Rational curves and characterizations of projective space

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In 1979, S. Mori gave an affirmative proof of Hartshorne’s conjecture, namely that the only smooth complex projective varieties having ample tangent bundle are projective spaces. Mori’s key observation is that when $TX$ is ample, $X$ is covered by rational curves, and he recovers the projective space by studying the behavior of (certain families of) such curves on $X$. These days, more systematic methods of examining the rational curves on a variety have been developed, and there are many results in the literature characterizing projective spaces and quadric hypersurfaces in terms of positivity properties of the tangent bundle.

For the first part of the talk, I will give an introductory survey of the most important techniques used to study families of rational curves and a few nice examples of how these techniques have been applied in practice. In the second part of the talk, I will discuss the current collection of results in the literature characterizing projective spaces and quadric hypersurfaces, as well as recent progress toward a conjecture of S. Kovacs that unifies these results.