Goal: Gain techniques for working with series which do not only have positive terms. Determine when an alternating series converges. Bound our error if we estimate an alternating series with a finite sum.

Let's start with a definition. An *alternating series* is a series whose terms are alternately positive and negative.

We can express the general term of (the sequence which defines) an alternating series as

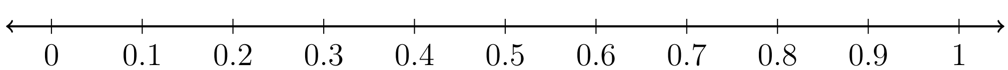
$$a_n = (-1)^n b_n \quad \text{or} \quad a_n = (-1)^{n-1} b_n$$

where $b_n > 0$. Notice that $b_n = |a_n|$.

1. We will determine if the alternating harmonic series $\displaystyle\sum_{n=1}^{\infty}(-1)^{n-1}\frac{1}{n}$ converges or diverges.

   (a) Write out the first five terms of the sequence that defines this series (that is, write out the first five terms of a_n).

   (b) Write out the first five terms of the sequence of partial sums for this series (that is, write out the first five terms of s_k). Plot (and label) them on the number line below.



   (c) If you continued this process for $s_6, \ldots, s_{10}$, where do you think they would fall?

   (d) Do you think that the sequence $\{s_k\}$ converges? Explain your thought process.

2. Consider the alternating series $\sum_{n=1}^{\infty}(-1)^n\left(1+\frac{1}{k}\right)$. Follow the same process outlined in problem 1 and make a conjecture about whether this series converges or diverges.

Leaning on your intuition for the two alternating series you've investigated, what are two things that are key for an alternating sequence to converge?

Alternating Series Test: If the alternating series $\sum_{n=1}^{\infty}(-1)^{n-1}b_n$ (where $b_n > 0$ for all n) satisfies

(i)

(ii)

then the series is convergent.

This is crazy! This is what we've wanted to be true since we started talking about series: the limit of the sequence is *almost* enough to determine if the series converges! All that's left to check is if we're consistently adding up "smaller" terms in absolute value. Easy, right?

3. Determine if $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^n 2n}{4n+1}$ converges or diverges with full justification.

4. Determine if $\displaystyle\sum_{n=1}^{\infty}(-1)^{n+1}\frac{n^2+1}{n^3-5}$ converges or diverges with full justification.

5. Here are some problems to work on to master alternating series. Unless otherwise stated, the instructions are "determine if the following series converge or diverge."

   (i) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{\ln(n+4)}$

   (ii) $\displaystyle\sum_{n=1}^{\infty} (-1)^{n+1} \frac{3n^2}{9n^2+9}$

   (iii) $\displaystyle\sum_{n=3}^{\infty} (-1)^{n} \frac{n}{\sqrt{n^3+2}}$

   (iv) $\displaystyle\sum_{n=1}^{\infty} (-1)^{n} \cos\left(\frac{\pi}{n}\right)$ and $\displaystyle\sum_{n=2}^{\infty} (-1)^{n} \sin\left(\frac{\pi}{n}\right)$

   (v) $\displaystyle\sum_{m=1}^{\infty} (-1)^{m} \cos(\pi m)$

   (vi) For which values of p does $\displaystyle\sum_{n=1}^{\infty} (-1)^{n} \frac{1}{n^p}$ converge?