Goal: Play with series!

1. Explain the difference between each pair of series.

   (a) $\sum_{i=1}^{\infty} a_i, \sum_{j=1}^{\infty} a_j$

   (b) $\sum_{i=1}^{\infty} a_i, \sum_{i=3}^{\infty} a_i$

   (c) $\sum_{i=1}^{\infty} a_i, \sum_{j=1}^{\infty} a_i$

2. Express each series in summation notation. Show that the series is convergent or that it is divergent. If it is convergent, find its sum.

   (a) $3 - 4 + \frac{16}{3} - \frac{64}{9} + \cdots$

   (b) $4 + 3 + \frac{9}{4} + \frac{27}{16} + \cdots$

   (c) $\sqrt{2} + \sqrt[3]{2} + \sqrt[4]{2} + \cdots$

3. Determine whether each series below is convergent or divergent.

   (a) (Homework 7) $\displaystyle\sum_{n=1}^{\infty} \frac{n(n+2)}{(n+3)^2}$

   (b) $\displaystyle\sum_{k=2}^{\infty} \cos\left(\frac{1}{k}\right)$

   (c) $\displaystyle\sum_{n=2}^{\infty} \frac{2}{n^2-1}$

(d) (Homework 7) $\sum_{i=1}^{\infty}(\cos 1)^i$

(e) $\sum_{n=1}^{\infty}\frac{2}{n^2+4n+3}$

(f) (Homework 7) $\sum_{n=1}^{\infty}\ln\left(\frac{n}{n+1}\right)$

4. For each of the series below, find the values of x for which the series converges. For those values of x, find the sum of the series.

   (a) $\displaystyle\sum_{n=0}^{\infty} \frac{x^n}{5^n}$

   (b) $\displaystyle\sum_{n=0}^{\infty} \frac{(x-4)^n}{3^n}$

   (c) $\displaystyle\sum_{k=0}^{\infty} \frac{(\sin x)^k}{2^k}$

5. ☕ If the nth partial sum of a series $\sum_{n=1}^{\infty} a_n$ is $s_n = \frac{n-1}{n+1}$ find a_n and $\sum_{n=1}^{\infty} a_n$.

6. ☕ If the nth partial sum of a series $\sum_{n=1}^{\infty} a_n$ is $s_n = 3 - n2^{-n}$ find a_n and $\sum_{n=1}^{\infty} a_n$.

7. ☕ Find the value of c if $\displaystyle\sum_{n=2}^{\infty}(1+c)^{-n} = 2$.

8. ☕ Suppose that a series $\sum a_n$ has positive terms and its partial sums s_n satisfy the inequality $s_n \leq 1000$ for all n. Does $\sum a_n$ converge or diverge? Prove your claim or justify that there is not enough information to do so.