Goal: Use slope fields to describe and/or sketch solutions to differential equations.

Let's start with a bit more terminology.

An *autonomous* differential equation of the form $y' = f(y)$ in which the independent variable is missing from the right side. In particular, this means that the slopes corresponding to different points with the same y coordinate must be equal.

Recall from yesterday that if a solution is constant, we call it an *equilibrium solution.* We classify equilibrium solutions as stable and unstable.

A *stable* equilibrium solution is one in which solutions that start "near" the equilibrium solution move toward the equilibrium solution.

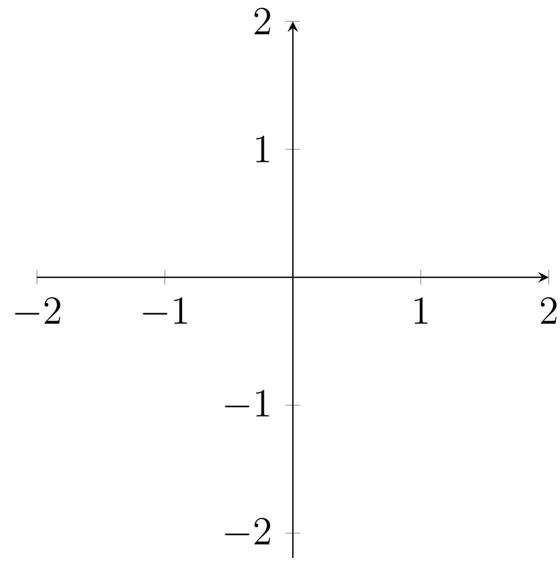
An *unstable* equilibrium solution is one in which solutions that start "near" the equilibrium solution move away from the equilibrium solution.

A *slope field* (or *direction field*) for a differential equation of the form $y' = F(x, y)$ is a sketch of short line segments of slope $F(x, y)$ drawn at several points (x, y).

Let's jump right in!

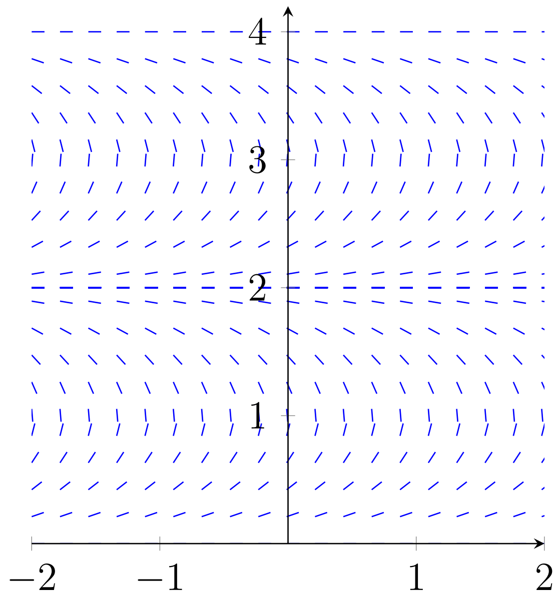
1. Sketch the solution curve of the differential equation $y' = x + y$ satisfying the initial condition $(0, 1)$.

   (a) Sketch the slope field for the differential equation $y' = x + y$



   (b) Sketch the solution curve through $(0, 1)$.

2. Below is a direction field for the differential equation $y' = \tan\left(\frac{\pi y}{2}\right)$.



(a) Sketch the graphs of the solutions that satisfy the given initial conditions.

$$y(0) = 1 \quad y(0) = 0.2 \quad y(0) = 2 \quad y(1) = 3$$

(b) Find and classify all of the equilibrium solutions.