Goal: Investigate how functions and their rates of change are related.

1. Consider the exponential function $y = 1.7e^{2t}$. What is the relationship between the instantaneous rate of change $\frac{dy}{dt}$ and t?

   What is the relationship between the instantaneous rate of change $\frac{dy}{dt}$ and y?

2. Consider the exponential function $y = 5^t$. What is the relationship between the instantaneous rate of change $\frac{dy}{dt}$ and t?

   What is the relationship between the instantaneous rate of change $\frac{dy}{dt}$ and y?

The equations that you just wrote are examples of differential equations. A *differential equation* is an equation that contains a function (usually unknown) and one or more of its derivatives. The *order* of a differential equation is the order of the highest derivative that occurs in the equation. Below is an example of a first order differential equation

$$y' = xy$$

where we interpret y as a function of x. We say a function f is a *solution* of a differential equation if when $y = f(x)$ and its derivatives are substituted into the equation, the equation is true. If a solution is constant, we call it an *equilibrium solution*. In our mini-example, $f(x)$ is a solution if

$$f'(x) = xf(x)$$

3. Verify that every member of the family of functions $y = \dfrac{1+ce^t}{1-ce^t}$ is a solution of the differential equation $y' = \dfrac{1}{2}(y^2 - 1)$

When we are asked to "solve" a differential equation, we want all possible solutions and we usually want to classify them into families. Unfortunately, there's no "one-size fits all" method to solving differential equations but don't fear! There are a few strategies we will employ as well as ways to approximate solutions graphically and numerically.

Often (especially in application problems), we are given an extra piece of data of the form $y(t_0) = y_0$, called an *initial condition.* This helps us pick a particular solution among all of our possible options.

4. Find a solution to the differential equation

$$y' = \frac{1}{2}(y^2 - 1)$$

that satisfies the initial condition $y(0) = 2$.

5. Create a differential equation that models the motion of a spring. Consider a vertical spring with a mass of m at the end.

   (a) What relation does Hooke's Law give us? (Let x represent the distance past the natural length of the spring.)

   (b) Let's look at force. How can we rewrite force using Newton's Second Law (force is mass times acceleration)?

   (c) How is this a differential equation? What is the order of the differential equation?

6. We will gain some more tools soon but for now, here are some rad differential equation problems for practice!

   (i) For what values of r does the function $y = e^{rx}$ satisfy the differential equation $2y'' + y' - y = 0$? If r_1 and r_2 are such values of r, show that every member of the family of functions $y = ae^{r_1 x} + be^{r_2 x}$ is also a solution.

   (ii) A population is modeled by the differential equation $\dfrac{dP}{dt} = 1.2P\left(1 - \dfrac{P}{4200}\right)$. For what values of P is the population increasing? Decreasing? What are the equilibrium solutions?