Yesterday we saw that work is given by the formula $W = \int_a^b f(x)dx$ where x is distance and $f(x)$ gives the force on an object at position x.

1. Suppose you are lifting a cable that weighs 2 lbs/ft to the top of Republic Plaza. Assuming the cable is 500 ft long, we want to determine how much work is done by lifting the entire cable onto the top of the building.

   Note: Republic Plaza is 714 feet tall, so the cable will be entirely off the ground at the start.

   (a) Suppose you have not started lifting the cable. What is the force you have to counteract in order to start moving the cable?

   (b) Suppose you have lifted 150 ft of the cable onto the roof. What is the force you have to counteract to continue moving the cable?

   (c) Suppose you have lifted x ft of the cable onto the roof. What is the force you have to counteract to continue moving the cable? (That is, what is $f(x)$?)

   (d) What will be the limits of integration on our integral? That is, suppose $f(x)$ is the force function, what values of x make sense in the context of the problem?

   (e) Suppose you have lifted the entire cable to the roof of the building. How much work was done?

2. A cable that weighs 2 lbs/ft is used to lift 800 lbs of coal up a mine shaft 500 ft deep. Find the work done.

   Hint: This problem is very similar to problem (1).

From physics, we know water pressure increases as water depth increases. In fact, the pressure exerted on an object in a liquid is given by the formula $P = \rho g d$ where

- ρ (pronounced "rho") is the mass density of the liquid. If the liquid is water, $\rho = 1000$ kg/m^3.
- g is the acceleration due to gravity. In the metric system, this is 9.8 m/sec^2.
- d is the distance below the surface of the liquid (in meters).

Warning: It is important to the note that the pressure on a particular point on an object does *not* depend on the object's orientation (whether the point is on the side/top/bottom of the object).
The units of pressure are Pascals (Pa) which are N/m^2. In particular, pressure is the force exerted by the liquid divided by the unit area.

3. A tank is 8 m long, 4 m wide, 2 m high, and contains kerosene with density 820 kg/m^3 to a depth of 1.5 m.

   (a) Find the hydrostatic pressure on the bottom of the tank.

   (b) Find the hydrostatic force on the bottom of the tank.

   Hint: From the comments above, we know pressure is force divided by area. This means $F = P \cdot A$.

(c) Find the hydrostatic force on one end of the tank.

Hint: Consider a thin horizontal slice at distance x from the top of the kerosene. What is the area of the end touching this slice? If the slice is thin, the pressure on all of that area is approximately constant. Turn this sum approximation into a integral by taking the limit as the number of slices approached infinity.

4. A trough is filled with a liquid of density 840 kg/m^3. Then ends of the trough are equilateral triangles with sides 8 m long and vertex at the bottom. Find the hydrostatic force on one end of the trough.

5. Here are some more problems involving work and pressure!

   (i) ☕ A heavy rope, 50 ft long, weighs 0.5 lbs/ft and hangs over the edge of a building 120 ft high.

      (a) How much work is done in pulling the rope to the top of the building?

      (b) How much work is done in pulling half of the rope to the top of the building?

   (ii) ☕ Suppose that 2 J of work is needed to stretch a spring from its natural length of 30 cm to a length of 42 cm.

      (a) How much work is needed to stretch the spring from 35 cm to 40 cm?

      (b) How far beyond its natural length will a force of 30 N keep the spring stretched?

(iii) ♨ An aquarium 2 m long, 1 m wide, and 1 m deep is full of water. Find the work needed to pump half of the water out of the aquarium. Use the fact that the density of water is 1000 kg/m^3.

(iv) ♨ A vertical, irregularly shaped plate is submerged in water. The table shows measurements of its width, taken at the indicated depths. Use Simpson's Rule to estimate the force of the water against the plate.

Depth (m)	2.0	2.5	3.0	3.5	4.0	4.5	5.0
Plate width (m)	0	0.8	1.7	2.4	2.9	3.3	3.6