Goal: Gain some intuition for the procedure of computing work using integrals.

In physics, the *force* on an object is defined as $F =$ (mass)(acceleration) which comes from Newton's Second Law of Motion (where acceleration is measured in the direction of motion). In SI metric units, force is measured in newtons ($\text{N} = \text{kg}\cdot\text{m/s}^2$). In the US Customary system, force is measured in pounds.

When moving an object, *work* is defined to be the product of the force on the object and the distance the object moves, that is, $W = Fd$ or work = (force)(distance). Force is measured in newton-meters (called joules and abbreviated J) or foot-pounds (ft-lb).

If the force and distance are constant, computing work is pretty easy. For example, compute the work don in moving a 1.2 kg book from the floor to put it on a desk that is 0.7 m high. (Hint: Start by computing the force on the book at any time.)

However, constant force and distance is a really special case and we'd prefer not to restrict ourselves to such a nice situation. Let's orient ourselves in a general case. Suppose we have an object moving along the x-axis from a to b and the force on that object is changing depending on where the object is according to some function $f(x)$. The work done in moving the object from a to b is given by

$$W = \int_a^b f(x)\,dx.$$

You can convince yourself of this by thinking of dx as an infinitesimal distance and we are adding up the work done $f(x)$ times the distance at each infinitesimal distance until we get the total work done.

1. When a particle is located a distance x meters from the origin, a force of $x^3 + x - 1$ newtons acts on it. How much work is done in moving it from $x = 1$ to $x = 3$?

Hooke's Law: The force required to maintain a spring stretched x units beyond its natural length is proportional to x:

$$f(x) = kx$$

where k is a positive constant (called the *spring constant*). Warning: There is something to be concerned about here—if x is too large, this doesn't hold but we will live in the world where we only look at close enough x.

2. A force of 10 lbs is required to hold a spring stretched 4 in. beyond its natural length. How much work is done in stretching it from its natural lenth to 6 in. beyond its natural length?

3. A particle is moved along the x-axis by a force that measures $10/(1+x)^2$ pounds at a point x feet from the origin. Find the work done in moving the particle from the origin to a distance of 9 ft.