Goal: Evaluate integrals involving polynomials in the denominator decomposing the product into factors that are easier to integrate.

A General Procedure

1. Before we even think about partial fraction decomposition, we need to make sure that we can! We need our integrand to be a rational function with a smaller degree in the numerator than the denominator. Otherwise, you will need to start with polynomial long division until you have a remainder of the desired form.

2. Factor the denominator as much as possible. (While you're at it, make sure nothing cancels in the numerator giving us a hole in our function—we'll learn how to deal with this when we talk about improper integrals.)

3. Identify what the terms in your decomposition look like. (Do you have all distinct roots in the denominator? Are some roots repeated? Do you have nonlinear terms?)

4. Multiply, expand, and collect like-terms.

5. Set coefficients equal and solve the system of equations.

6. Return to your integral and you should be able to integrate each piece.

1. Evaluate $\displaystyle\int_2^3 \frac{1}{x^2-1}\,dx$.

2. Evaluate $\displaystyle\int \frac{6-7x}{(x-1)(x+4)^2}\,dx$.

3. Evaluate $\displaystyle\int \frac{5x+1}{(2x+1)(x-1)}\,dx$.

4. Evaluate $\displaystyle\int \frac{10}{(x-1)(x^2+9)}\, dx$.

5. Evaluate $\displaystyle\int \frac{1}{x^3+2x^2+x}\, dx$.

6. Here are some integrals that you can add to your "I've got this!"-list.

   (i) $\displaystyle\int \frac{x}{x^2+x-2}\,dx$

   (ii) $\displaystyle\int \frac{x^3+x^2+2x+1}{(x^2+1)(x^2+2)}$

   (iii) $\displaystyle\int_0^1 \frac{x-4}{x^2-5x+6}$

(iv) $\displaystyle\int \frac{x^2 - x + 6}{x^3 + 3x}\,dx$

(v) $\displaystyle\int \frac{r^2}{r+4}$

(vi) $\displaystyle\int_9^{16} \frac{\sqrt{x}}{x-4}\,dx$ Hint: It may be useful to do a substitution to express the integrand as a rational function before evaluating the integral.

(vii) $\displaystyle\int \frac{x^3 + 3x^2 - 4x - 9}{x^2 - x - 2}\, dx$

(viii) Evaluate $\displaystyle\int \frac{2x+4}{(x+6)(x-2)}\, dx$ using two different methods.