Goal: Evaluate integrals of products of trigonometric functions.

Recall that on the project from last Thursday, you computed various integrals of the form

$$\int \big(\sin(x)\big)^m \big(\cos(x)\big)^n \, dx.$$

There are several key facts that help us work with the integrand until it is in a form we can compute. List them below and add to this list as you work through problems.

If n is odd, how would you approach the problem? (Try computing a few of the early problems below if you have forgotten!)

If m is odd, how would you approach the problem?

What would you try if both m and n are even?

How can we adapt these strategies to evaluate integrals of the form

$$\int \big(\sec(x)\big)^m \big(\tan(x)\big)^n \, dx?$$

1. (Review from Thursday) Evaluate $\displaystyle\int_{\pi/2}^{3\pi/4} \big(\sin(x)\big)^5 \big(\cos(x)\big)^3 \, dx.$

2. Evaluate $\displaystyle\int \big(\tan(y)\big)^3 \sec(y) \, dy.$

3. Evaluate $\displaystyle\int_0^{2\pi} \big(\cos(6\theta)\big)^2 \, d\theta.$

4. Evaluate $\int \sec(x)\ dx$.

   Hint: Multiply the integrand by the (not at all obvious) "fancy 1" $\dfrac{\sec(x)+\tan(x)}{\sec(x)+\tan(x)}$.

5. Evaluate $\int \big(\sec(x)\big)^3\ dx$. Hint: Try integrating by parts using $u = \sec(x)$.

6. Evaluate $\int \tan(x)\ dx$.

7. Evaluate $\int \big(\tan(x)\big)^3\ dx$.

8. Here is a list of integrals to sharpen your shiny new trigonometric integration tool:

   (i) $\int \sin(8x)\cos(5x)\,dx$ Hint: Try using sum and difference formulas.

   (ii) $\int \big(\sin(3x)\big)^8\big(\cos(3x)\big)^5\,dx$

   (iii) $\int \big(\tan(x)\big)^7\big(\sec(x)\big)^3\,dx$

(iv) $\displaystyle\int \cot(x)\,dx$

(v) $\displaystyle\int \big(\cot(x)\big)^3\,dx$

(vi) $\displaystyle\int \csc(x)\,dx$

(vii) $\displaystyle\int \big(\csc(x)\big)^3\,dx$

(viii) ☕ $\int \big(\tan(x)\big)^5 \big(\cos(x)\big)^5\, dx.$

(ix) ☕ $\int \big(\cot(x)\big)^5 \csc(x)\ dx$