Goal: Find an expression for $\int_a^b f(x)g'(x)\ dx$.

Recall that $\int_a^b f'(x)dx = \int_a^b \frac{d}{dx}[f(x)]dx = f(b) - f(a)$. (This fact is called the Fundamental Theorem of Calculus!) Using this fact, what does the following integral equal?

$$\int_a^b \frac{d}{dx}\left[f(x)g(x)\right]\ dx =$$

Let's look a little closer at the integral we started with. Using the product rule for derivatives, rewrite the integrand. (You should be left with just one integral over a sum of functions.)

$$\int_a^b \frac{d}{dx}\left[f(x)g(x)\right]\ dx = \int_a^b \Bigg(\qquad\qquad\qquad\qquad\qquad\qquad \Bigg) dx$$

Now, integrals distribute over sums so let's rewrite our integral one more time but now as the sum of two integrals.

$$\int_a^b \frac{d}{dx}\left[f(x)g(x)\right]\ dx = \int_a^b \Bigg(\qquad\qquad\qquad\qquad \Bigg) dx + \int_a^b \Bigg(\qquad\qquad\qquad\qquad \Bigg) dx$$

From the first equation on this page, we computed the left-hand side of this equation so rewrite the full equation below. (It should have no integrals on the left and two integrals on the right—one that we are looking for!)

Remember what we are looking for: an expression for $\int_a^b f(x)g'(x)\ dx$. Rearrange the previous equation for this term and you have the formula that we call "Integration by Parts."

$$\int_a^b f(x)g'(x)\ dx =$$

You can also do this for indefinite integrals!

1. Solve $\int_0^1 xe^x dx$.

2. Evaluate $\int_0^{\pi} x\cos(x)dx$.

3. Evaluate $\int_1^5 x^2 e^{2x} dx$.

4. Evaluate $\int \sin(x)e^x dx$.

5. Here is a list of integrals to sharpen your shiny new Integration-by-Parts tool:

   (i) $\displaystyle\int 2x^3 \cos\left(\frac{x}{3}\right) dx$

   (ii) $\displaystyle\int (2x-3)^2 e^{\frac{x}{2}}\, dx$

(iii) $\int \ln(x)dx$

(iv) $\int 2x^4 \ln(x)dx$

(v) $\int \arcsin(x)dx$

(vi) $\int x^3 3^x dx$

(vii) $\int \frac{\ln(x)}{x^2} dx$

(viii) $\int (\ln(x))^2 dx$

(ix) $\int \cos(2x)e^{3x}dx$

(x) $\int \arctan(4t)\ dt$

(xi) ♨ Prove that $\displaystyle\int \big(\ln(x)\big)^n \, dx = x\big(\ln(x)\big)^n - n\int \big(\ln(x)\big)^{n-1} \, dx.$

Hint: It may be helpful to first evaluate $\displaystyle\int \big(\ln(x)\big)^2 \, dx.$