Goal: Gain and utilize tools to determine if an improper integral converges or diverges.

In the last project, you learned about two tests that will help you to determine if an improper integral converges or diverges.

Comparison Test for Integrals: If f and g are continuous functions with $f(x) \geq g(x) \geq 0$ for $x \geq a$, then

(a) If $\int_a^\infty f(x)\,dx$ is convergent, then

(b) If $\int_a^\infty g(x)$ is divergent, then

Limit Comparison Test for Integrals: If $f(x)$ and $g(x)$ are positive, continuous functions and $\lim_{x\to\infty} \frac{f(x)}{g(x)} = c$, where $0 < c < \infty$, then

Here is a list of integrals that you can now decide if they converge or diverge and determine the corresponding value. Remember to *explicitly* check that all of the hypotheses for a theorem/test hold before you use them, even if they are obvious.

(i) $\int_3^\infty \frac{x^2}{x^4+4}\,dx$

(ii) $\int_0^\infty \frac{1}{(x-2)^2}\,dx$

(iii) $\displaystyle\int_2^4 \frac{2}{x\sqrt{x^2-4}}dx$

(iv) $\displaystyle\int_0^\infty \sin\left(\frac{x}{2}\right)$

(v) $\displaystyle\int_0^2 \frac{1}{4-x^2}dx$

(vi) $\displaystyle\int_0^\infty e^{-2x}\sin(x)dx$

(vii) ☕ $\displaystyle\int_3^6 \frac{1}{(x-5)^2}dx$

(viii) ☕ $\displaystyle\int_1^\infty \frac{1-e^{-x}}{x}\,dx$

(ix) ☕ For what values of p does $\displaystyle\int_{-1}^1 \frac{1}{x^p}\,dx$ converge?