Goal: Gain intuition and familiarity with parametric curves.

Let's start with an example.

1. Suppose a particle is traveling for 6 seconds starting at $t = 0$ (measured in seconds). At time t, the particle is at position $\big(x(t), y(t)\big)$ where $x(t) = \sin(\pi t)$ and $y(t) = \cos(\pi t)$.

   (a) Using a table of values, sketch the path of the particle. Use arrows to indicate the direction of travel.

   (b) Describe the path of the particle.

   (c) Just using a table of values is not enough to prove that the particle takes exactly the path you've described above. To prove that, we need to find an equation that gives the position of the particle without using t. Try to find such an equation. It may be useful to use the identity $(\sin\theta)^2 + (\cos\theta)^2 = 1$.

Let's formalize a few of the ideas in the previous problem. If x and y are given by functions of another variable, t, we say that x and y are *parametrized* by the *parameter* t and we call $x = f(t)$ and $y = g(t)$ *parametric equations*. We sometimes use *vector notation* which is denoted $\langle f(t), g(t)\rangle$. This gives us a curve in two dimensions and you could (and will!) do the same in three dimensions.

Generally, we want to obtain an expression for the curve generated by a parametric equation using only x and y with no mention of the parameter t (called a *Cartesian equation*). We call that *eliminating the parameter*.

We often interpret a set of parametric equations as a position of a particle at time t but this is not always the case.

Why would we use parametric equations? Well, in our example above, the path of the particle doesn't pass the vertical line test so we can never express the curve as a function $y = f(x)$ so a lot of our calculus skills don't apply. How can we take a derivative of something that isn't a function? Luckily, our parameterization will allow us to do exactly that over the next few days. For now, let's get back to just drawing a few more examples.

2. Sketch the parametric curve given by

$$\begin{cases} x(t) = t^2 \\ y(t) = 2t - 1 \end{cases} \quad -1 \leq t \leq 2 \quad \text{or} \quad \langle t^2, 2t-1\rangle$$

Also, eliminate the parameter to find a Cartesian equation of the curve.

3. Here are a couple additional problems that will help you master parametric equations!

   (i) Sketch the parametric curve given by

$$\begin{cases} x(t) = \cos(t) \\ y(t) = \sin(4t) \end{cases} \quad 0 \leq t \leq 2\pi \quad \text{or} \quad \langle \cos(t), \sin(4t) \rangle$$

(ii) Suppose that the position of one particle at time t is given by

$$x_1 = 3\sin t \quad y_1 = 2\cos t \quad 0 \le t \le 2\pi$$

and the position of the second particle is given by

$$x_2 = -3 + \cos t \quad y_2 = 1 + \sin t \quad 0 \le t \le 2\pi$$

(a) Graph the paths of both particles.
(b) How many intersection points are there?
(c) Are any of these points of intersections collision points? (That is, are they particles ever at the same place at the same time?) If so, find the collision points.
(d) Lastly, describe what happens if the path of the second particle is given by

$$x_3 = 3 + \cos t \quad y_3 = 1 + \sin t \quad 0 \le t \le 2\pi$$

instead.