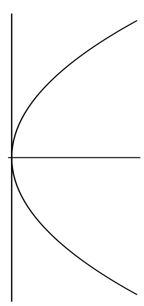
	Parabola Equations	Vertex	Focus	Directrix
1)	$(y-k)^2 = 4p(x-h)$	(h,k)	$(h+p,k)$	$x = h-p$
2)	$(y-k)^2 = -4p(x-h)$	(h,k)	$(h-p,k)$	$x = h+p$
3)	$(x-h)^2 = 4p(y-k)$	(h,k)	$(h,k+p)$	$y = k-p$
4)	$(x-h)^2 = -4p(y-k)$	(h,k)	$(h,k-p)$	$y = k+p$

1)



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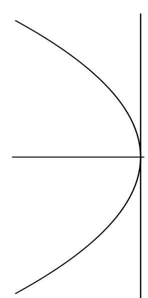

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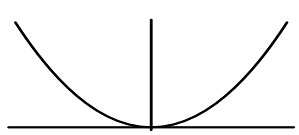

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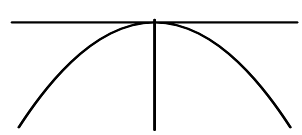
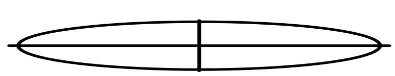


Ellipse Equations	Center	Major Axis	Minor Axis	Focus Points
1) $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$	(h, k)	$2a$	$2b$	$(h+c, k)$ $(h-c, k)$
2) $\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$	(h, k)	$2a$	$2b$	$(h, k+c)$ $(h, k-c)$

Remark: The major axis is the longer of the two axes and the minor axis is the shorter of the two. Equation 1 indicates that the major axis is on a horizontal line, while equation 2 indicates that the major axis is on a vertical line. Also, the equations $c^2 = a^2 - b^2$ and $e = \frac{c}{a}$, where e is the eccentricity, hold.

1)



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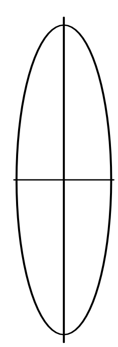
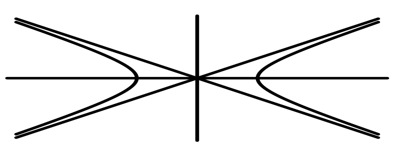


	Hyperbola Equations	Center	Focus Points	Vertices	Asymptotes
1)	$\dfrac{(x-h)^2}{a^2} - \dfrac{(y-k)^2}{b^2} = 1$	(h,k)	$(h+c,k)$	$(h+a,k)$	$y-k = \dfrac{b}{a}(x-h)$
			$(h-c,k)$	$(h-a,k)$	$y-k = -\dfrac{b}{a}(x-h)$
2)	$\dfrac{(y-k)^2}{a^2} - \dfrac{(x-h)^2}{b^2} = 1$	(h,k)	$(h,k+c)$	$(h,k+a)$	$y-k = \dfrac{a}{b}(x-h)$
			$(h,k-c)$	$(h,k-a)$	$y-k = -\dfrac{a}{b}(x-h)$

Remark: Equation 1 indicates that the focus points are on a horizontal line, while equation 2 indicates that the focus points are on a vertical line. Also, the equation $c^2 = a^2 + b^2$ holds; note that this is a different equation than the one for the ellipse. The equation $e = \frac{c}{a}$, where e is the eccentricity, holds (same as in the ellipse).

1)



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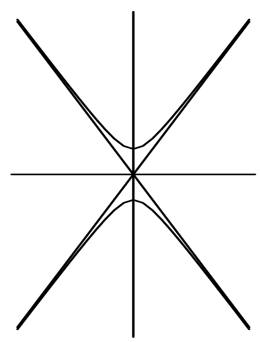


3) Find the equation and sketch the graph of the parabola with vertex $V(2,3)$ and focus $F(2,1)$.

Solution:

Solving for p, we get $p = 3 - 1 = 2$ so the equation is $(x - 2)^2 = -4(2)(y - 3)$. The directrix is the line $y = 5$.

8) Find the equation and sketch the graph of the parabola with focus $F(1,-1)$ and directrix $x = 3$.

Solution:

The distance between the focus and the directrix is $3 - 1 = 2$ and p is half of this, so we get $p = 1$ and the equation is $(y + 1)^2 = -4(x - 2)$. The vertex is (2, -1).

11) Sketch the parabola $y^2 = 12x$. Show and label its vertex, focus, axis, and directrix.

Solution:

The equation $y^2 = 12x$ implies $y^2 = 4 \cdot 3(x)$. So, the vertex is at (0,0); focus at (3,0); and the directrix is the line $x = -3$. The axis of symmetry is the x-axis.

17) Sketch the parabola with the given equation. Show and label its vertex, focus, axis, and directrix.

$4x^2 + 4x + 4y + 13 = 0$

Solution:

Completing the square, we get:

$$\begin{aligned} 4(x^2 + x + \tfrac{1}{4}) &= -4y - 13 + 1 \\ (x + \tfrac{1}{2})^2 &= -y - 3 \\ (x + \tfrac{1}{2})^2 &= -(y + 3) \\ (x + \tfrac{1}{2})^2 &= -4(\tfrac{1}{4})(y + 3) \end{aligned}$$

So $p = \frac{1}{4}$; the vertex is at $(-\frac{1}{2}, -3)$; focus at $(-\frac{1}{2}, -\frac{13}{4})$; and the directrix is the line $y = -\frac{11}{4}$. The axis of symmetry is the line $x = -\frac{1}{2}$.

21) Find an equation of the ellipse.

Foci $(0, 8)$ and $(0, -8)$; major semiaxis 17.

Solution:

Note that when you plot the foci, both points lie on a vertical line with midpoint (0,0). That means the center is (0,0) and the ellipse equation has the form $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$.

Now the distance between the foci is 16 and the value of c is half this distance, so $c = 8$.

The major semiaxis is 17, so $a = 17$.

Using the formula $c^2 = a^2 - b^2$, we get $b = 15$.

And so the equation of the ellipse is $\frac{x^2}{15^2} + \frac{y^2}{17^2} = 1$.

26) Find an equation of the ellipse.

Center $(0, 0)$; horizontal minor axis is 10; eccentricity $\frac{1}{2}$.

Solution:

The problem tells us that the center is (0,0) and the minor axis is horizontal; that gives us an ellipse equation of the form $\dfrac{x^2}{b^2} + \dfrac{y^2}{a^2} = 1$.

Since the value of the minor axis is 10 and it's horizontal, we have that $b = 5$.

Using the equation $e = \frac{c}{a}$ and given that the eccentricity, e, is $\frac{1}{2}$, we get that $a = 2c$.

Using the formula $c^2 = a^2 - b^2$ and substituting $2c$ for a, we get $c = \sqrt{4c^2 - 25}$. Solving for c, we get $c = \frac{5}{3}\sqrt{3}$.

Using the equation $e = \frac{c}{a}$ again and substituting $\frac{5}{3}\sqrt{3}$ for c, we get $a = \frac{10}{3}\sqrt{3}$.

And so the equation of the ellipse is $\dfrac{x^2}{5^2} + \dfrac{y^2}{(\frac{10}{3}\sqrt{3})^2} = 1$ or $\dfrac{x^2}{25} + \dfrac{3y^2}{100} = 1$.

37) Sketch the graph. Indicate center, foci and lengths of axes.

$9x^2 + 4y^2 - 32y + 28 = 0$

Solution:

Completing the square, we get:

$$\begin{aligned}
9x^2 + 4(y^2 - 8y + 16) &= -28 + 64 \\
9x^2 + 4(y-4)^2 &= 36 \\
\tfrac{9x^2}{4} + (y-4)^2 &= 9 \\
\tfrac{x^2}{4} + \tfrac{(y-4)^2}{9} &= 1
\end{aligned}$$

So we have $a = 3$, $b = 2$ and $c = \sqrt{9-4} = \sqrt{5}$. The center is at (0,4). The foci are at $(0, 4+\sqrt{5})$ and $(0, 4-\sqrt{5})$. The major axis is vertical of length 6. The minor axis is horizontal of length 4.

38) Sketch the graph. Indicate center, foci and lengths of axes.

$2x^2 + 3y^2 + 12x - 24y + 60 = 0$

Solution:

Completing the square, we get:

$$\begin{aligned} 2(x^2+6x+9)+3(y^2-8y+16) &= -60 + 18 + 48 \\ 2(x+3)^2 + 3(y-4)^2 &= 6 \\ \tfrac{(x+3)^2}{3} + \tfrac{(y-4)^2}{2} &= 1 \end{aligned}$$

So we have $a = \sqrt{3}$, $b = \sqrt{2}$ and $c = \sqrt{3-2} = 1$. The center is at (-3,4). The foci are at $(-4, 4)$ and $(-2, 4)$. The major axis is horizontal of length $2\sqrt{3}$. The minor axis is vertical of length $2\sqrt{2}$.

42) Find an equation of the hyperbola.

Vertices $(3, 0)$ and $(-3, 0)$; asymptotes $y = \frac{3}{4}x$ and $y = -\frac{3}{4}x$.

Solution:

The vertices given above tell us the hyperbola has the form $\dfrac{(x-h)^2}{a^2} - \dfrac{(y-k)^2}{b^2} = 1$ because the vertices are on a horizontal line.

They also tell us that $a = 3$ and that the center is (0,0).

The asymptotes tell us that $\frac{b}{a} = \frac{3}{4}$, which implies $b = \frac{9}{4}$.

And so the equation of the hyperbola is $\dfrac{x^2}{3^2} - \dfrac{y^2}{(\frac{9}{4})^2} = 1$ or $\dfrac{x^2}{3^2} - \dfrac{16y^2}{81} = 1$.

54) Sketch the graph. Indicate center, foci and asymptotes.

$x^2 - 2y^2 + 4x = 0$

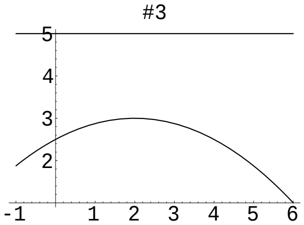
Solution:

Completing the square, we get:

$$\begin{aligned} x^2 + 4x + 4 - 2y^2 &= 4 \\ (x+2)^2 - 2y^2 &= 4 \\ \frac{(x+2)^2}{4} - \frac{y^2}{2} &= 1 \end{aligned}$$

So we have $a = 2, \quad b = \sqrt{2}$ and $c = \sqrt{4+2} = \sqrt{6}$. Note that we used the formula $c^2 = a^2 + b^2$ to compute c which differs from the formula for c in the ellipse equation. The center is at (-2,0). The foci are at $(-2+\sqrt{6}, 0)$ and $(-2-\sqrt{6}, 0)$. The vertices are at (-4,0) and (0,0) and the asymptotes are $y = \frac{\sqrt{2}}{2}(x+2)$ and $y = -\frac{\sqrt{2}}{2}(x+2)$.

#3



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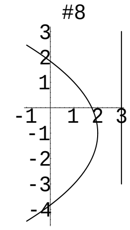

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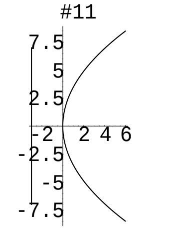

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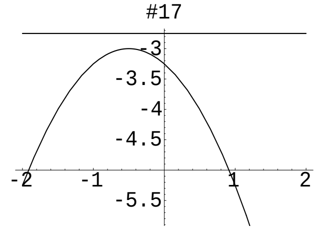

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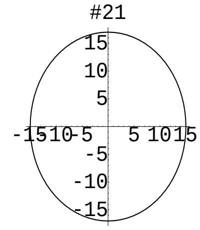

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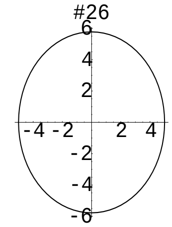

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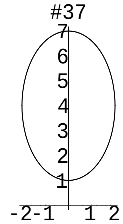

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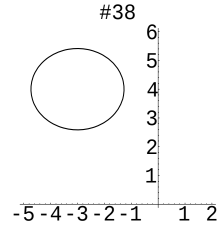

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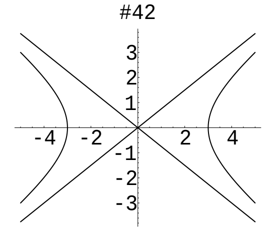

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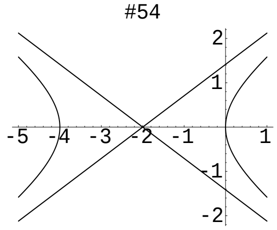

78) The orbit of Mercury is an ellipse of eccentricity $e = 0.206$. Its maximum and minimum distances from the Sun are 0.467 and 0.307 AU, respectively. What are the major and minor semiaxes of Mercury's orbit? Does "nearly circular" accurately describe the orbit?

Solution:

We'll take the sum of the max and min distances from the Sun to be the length of the major axis, so we have:
$2a = 0.467 + 0.307 = 0.774$ which gives us $a = 0.387$

Using the formula $e = \frac{c}{a}$ and the given value of 0.206 for e, we get:
$c = (0.206)(0.387) = 0.079722$

Re-writing the formula $c^2 = a^2 - b^2$ for b, we get:
$b = \sqrt{a^2 - c^2} \approx 0.3787$

So, the major axis length is $2a = 0.7740$ and the minor axis length is $2b = 2(0.3787) = 0.7574$.

Because the eccentricity is pretty close to zero ($e = 0.206$), we can refer to Mercury's orbit as nearly circular.

90) Two radio signaling stations at A and B lie on an east-west line, with A 100 mi west of B. A plane is flying west on a line 50 mi north of the line AB. Radio signals are sent (traveling at 980 ft/μs) simultaneously from A and B, and the one sent from B arrives at the plane 400 μs before the one sent from A. Where is the plane?

Solution:

If $|AP|$ denotes the distance from station A to the plane, then we have the equation

(1) $|AP| = 980 \cdot T_A$

where T_A is the time it takes for station A's signal to reach the plane.

Similarly, we have the equation

(2) $|BP| = 980 \cdot T_B$

where T_B is the time it takes for station B's signal to reach the plane.

Because A's signal took 400 μs longer to reach the plane, we have the equation

(3) $T_A = 400 + T_B$

Re-writing equation (1) for T_A; replacing T_A by $400 + T_B$; and replacing T_B by $\frac{|BP|}{980}$ we get

$\frac{|AP|}{980} = T_A = 400 + T_B = 400 + \frac{|BP|}{980}$

which implies

$|AP| - |BP| = 980 \cdot 400 = 392,000$ ft

Now, a hyperbola is actually formed by all the points P satisfying the equation $|AP| - |BP| = 2a$, so stations A and B are actually the focus points of this hyperbola and the plane is on one of the arcs. We have the equation

$$|AP| - |BP| = 2a$$

Since $|AP| - |BP| = 392,000$ ft, $2a = 392,000$ ft and solving for a gives us $a = 196,000$ ft ≈ 37 mi

Since A and B are the focus points and their coordinates are $(-50, 0)$ and $(50, 0)$ respectively, we have $c = 50$ mi

Using the equation $c^2 = a^2 + b^2$ to compute b, we get

$$b \approx 176861 \text{ ft} \approx 33 \text{ mi}$$

To recap, we have $a \approx 37$, $b \approx 33$, and $c = 50$ all in miles. The hyperbola equation is then

$$\frac{x^2}{1378} - \frac{y^2}{1122} = 1 \tag{4}$$

Equation (4) gives us the x and y coordinates of the plane.

Since the plane is flying on a line 50 mi north of station B, we can replace y by 50 in equation (4) and solve for x. Doing that gives us

$x^2 \approx 4448$ mi, which implies $x \approx 67$ mi

So, the plane is at the point (67 mi, 50 mi) in our coordinate system, or 17 miles east/50 miles north of station B.