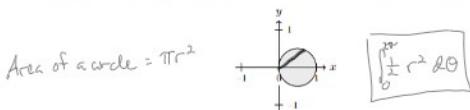
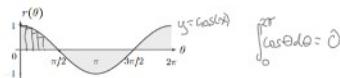


HW 8

3. Lee is working to find the area enclosed by the polar curve $r(\theta) = \cos(\theta)$ from $0 \leq \theta \leq 2\pi$. Lee graphed the areas of polar curve on the Cartesian xy -plane using polar coordinates



and the graph of r as a function of θ



When looking at the both areas, Lee notices that there is equal amounts of area above and below the horizontal axis of both graphs. Lee argues that because of this the area enclosed by the polar curve $r(\theta) = \cos(\theta)$ from $0 \leq \theta \leq 2\pi$ is equal to zero. Explain if Lee's reasoning is valid or not.

$$\text{Area enclosed by polar eq: } \frac{1}{2} \int_0^{2\pi} (r(\theta))^2 d\theta = \frac{1}{2} \int_0^{2\pi} \cos^2 \theta d\theta$$

HW 9

3. Alex was working to find the centroid for a lamina with constant density $\rho = 2$ and bounded by the given curves:

$$y = x^2, x + y - 2, y = 0$$

Alex remembered from class that the y -component for the centroid being

\bar{y} = \frac{1}{A} \int_a^b \frac{1}{2} (f(x))^2 dx

$$\bar{y} = \frac{1}{A} \int_a^b \frac{1}{2} (f(x))^2 dx$$

Alex knows the A represents the area of the lamina, but is unsure about the $f(x)$ term.
 ~~Is it $f(x)$ comes from, or $f(x)$ the entire area bounded by $f(x)$ in this formula?~~
 ~~R is the center of a vertical strip has an x -value, so we can substitute x into $f(x)$ to get the center.~~

Find the y -component for the problem Alex is working on, while helping Alex by explaining what each part of the formula from class represents:

Multiplying by the ρ inside of the integral, which has the same boundary function $f(x)$.

Vertical strips allow us to compute the location of the center.
 $\frac{1}{2} f(x)$ is the center of a vertical strip between $y=0$ and $y=f(x)$

$$A = \int_0^1 x^4 dx + \int_0^1 (-x+2) dx$$

$$A = \frac{x^5}{5} \Big|_0^1 + \left(-\frac{x^2}{2} + 2x \right) \Big|_0^1 = \frac{1}{5} + \left(2 + 2 - \left(-\frac{1}{2} + 2 \right) \right) = \frac{1}{5} + \frac{1}{2} = \frac{7}{10}$$

$$\bar{y} = \frac{1}{A} \int_a^b \frac{1}{2} (f(x))^2 dx = \frac{2}{7} \int_0^1 \frac{1}{2} (x^4)^2 dx = \frac{2}{7} \left[\int_0^1 (x^4)^2 dx + \int_0^1 (-x+2)^2 dx \right]$$

$$= \frac{2}{7} \left[\int_0^1 x^8 dx + \int_0^1 (-x+2)^2 dx \right] \dots$$

HW 9

1. Determine whether each sequence below converges or diverges. If the sequence converges, find its limit. Carefully show your work!

$$(a) a_n = \sqrt[n]{7^{1+2n}}$$

$$S(n) = (1 + \frac{2}{n})^{2n}$$

$$(b) b_n = \frac{(\ln n)^3}{n}$$

$$(d) d_n = \left(1 + \frac{2}{n}\right)^{3n}$$

$$(c) c_n = \frac{n!}{e^n}$$

$$(e) e_n = \frac{\sqrt{3+9n^2}}{3n^2+4}$$

$$\lim_{n \rightarrow \infty} \frac{\sin(2n)}{1+\sqrt{n}}$$

$$\frac{-1 \leq \sin(2n) \leq 1}{1+\sqrt{n}} \leq \frac{1}{1+\sqrt{n}}$$

$$\lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \sqrt[2n]{7^{1+2n}}$$

$$= \lim_{n \rightarrow \infty} (7^{1+2n})^{1/n}$$

$$= \lim_{n \rightarrow \infty} (7^{1/(1+2)})$$

$$= \lim_{n \rightarrow \infty} (7^{1/3})$$

$$= 7^2 \lim_{n \rightarrow \infty} (7^{1/n})$$

2. Determine whether each series below is convergent or divergent. If the series is convergent, find its sum. Carefully show your work, and justify any tests that you use.

$$(a) \sum_{k=1}^{\infty} \frac{(k+1)(3k-1)}{(k+3)^2}$$

$$(c) \sum_{k=1}^{\infty} \cos(5)^k$$

$$(b) \sum_{k=1}^{\infty} \ln\left(\frac{k}{k+1}\right)$$

$$(d) \sum_{k=3}^{\infty} \frac{1}{k^2+k}$$

$$\lim_{n \rightarrow \infty} \frac{-1}{1+\sqrt{n}} \leq \lim_{n \rightarrow \infty} \frac{\sin(2n)}{1+\sqrt{n}} \leq \lim_{n \rightarrow \infty} \frac{1}{1+\sqrt{n}}$$

$$\text{So } \lim_{n \rightarrow \infty} \frac{\sin(2n)}{1+\sqrt{n}} = 0 \text{ by the squeeze thm}$$

HW 5

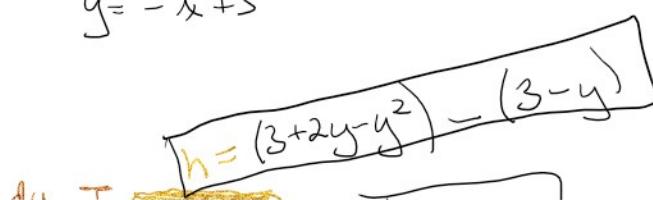
4. Use the method of cylindrical shells to find the volume generated by rotating the region bounded by the curves $x = 3 + 2y - y^2$ and $x + y = 3$ about the x -axis. Sketch the region and a typical shell.

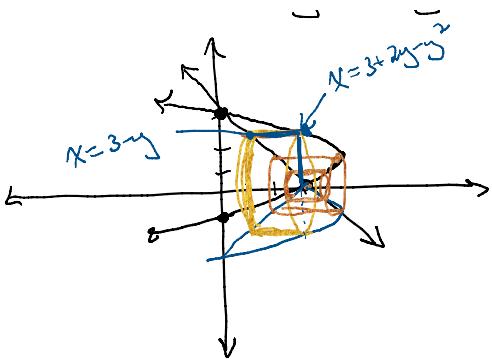
$$x = -(y^2 - 2y - 3)$$

$$x = -(y-3)(y+1)$$

$$y = -x + 3$$

$$h = (3+2y-y^2) - (3-y)$$





$$h = (3+2y - y^2) - (3-y)$$

$$r = y$$

SA of a cylindrical shell = $h \cdot 2\pi r$

$$SA = [(3+2y - y^2) - (3-y)] 2\pi y$$

$$\text{Volume} = dy \cdot SA$$

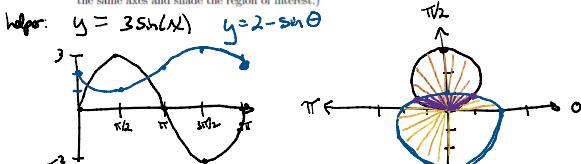
$$Vol = [By^2 + 3y] y 2\pi dy$$

$$2\pi \int_0^3 (-y^3 + 3y^2) dy = 2\pi \left(\frac{-y^4}{4} + y^3 \Big|_0^3 \right)$$

$$= 2\pi \left(\frac{-81}{4} + 27 \right)$$

HW 8

5. Find the area of the region that lies inside the curve whose polar equation is $r = 3\sin\theta$ and outside the curve whose polar equation is $r = 2 - \sin\theta$. (Hint: First, sketch the two curves on the same axes and shade the region of interest.)



$$3\sin(\theta) = 2 - \sin\theta$$

$$\sin\theta = \pm \frac{1}{2}$$

QI: $\frac{\pi}{6}$ QII: $\frac{5\pi}{6}$

$$A = \frac{1}{2} \int_a^b (f(\theta))^2 d\theta$$

$$A = 2 \left[\int_0^{\pi/6} (3\sin\theta)^2 d\theta + \int_{\pi/6}^{5\pi/6} (2 - \sin\theta)^2 d\theta \right]$$

$$= \int_0^{\pi/6} + \int_{\pi/6}^{5\pi/6} + \int_{5\pi/6}^{\pi}$$