Review

Monday, February 7, 2022 10:38 AM

① $\displaystyle\int \frac{5}{x^2\sqrt{x^2+4}}\,dx$

$a^2+b^2=c^2 \rightarrow c=\sqrt{a^2+b^2}$

$\tan\theta = \frac{x}{2} \rightarrow x = 2\tan\theta$

$dx = 2\sec^2\theta\, d\theta$

$\cos\theta = \frac{2}{\sqrt{x^2+4}} \rightarrow \sqrt{x^2+4} = \frac{2}{\cos\theta}$

$\sqrt{x^2+4} = 2\sec\theta$

$\sin\theta = \frac{x}{\sqrt{x^2+4}}$

$$= \int \frac{5}{(2\tan\theta)^2\, \cancel{2}\cancel{\sec\theta}}\, \cancel{2}\sec^{\cancel{2}}\theta\, d\theta$$

$$= \int \frac{5\sec\theta}{4\tan^2\theta}\, d\theta$$

$$= \int \frac{5\cdot\frac{1}{\cos\theta}}{4\cdot\frac{\sin^2\theta}{\cos^2\theta}}\, d\theta = \int \frac{5\cos\theta}{4\sin^2\theta}\, d\theta = \frac{5}{4}\int \frac{\cos\theta}{\sin^2\theta}\, d\theta$$

$u = \sin\theta$

$du = \cos\theta\, d\theta$

$$= \frac{5}{4}\int \frac{1}{u^2}\, du = -\frac{5}{4}u^{-1} + C = \frac{-5}{4\sin\theta} + C = \frac{-5}{4\frac{x}{\sqrt{x^2+4}}} + C = \boxed{\frac{-5\sqrt{x^2+4}}{4x} + C}$$

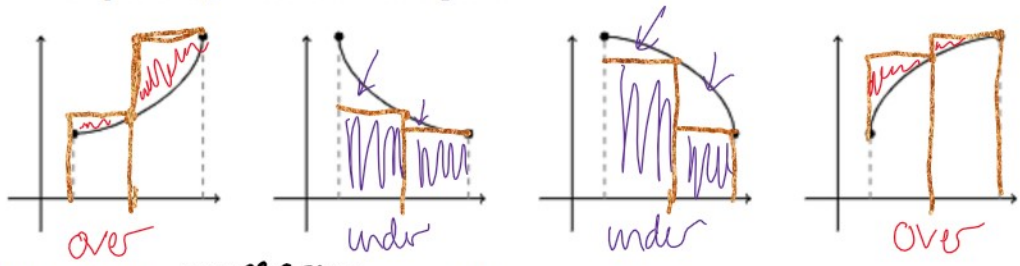
$$\frac{5}{4}\int u^{-2}\, du = \frac{5}{4}\frac{u^{-1}}{-1} + C$$

② Does the following limit converge or diverge? Justify completely.

$$\int_3^\infty \frac{\cos^2(x)}{x^2}\, dx$$

③

b. Examples of R_n. Please draw rectangles for $n = 2$.



When $f(x)$ is increasing, R_n is an overestimate.

When $f(x)$ is decreasing, R_n is an underestimate.

c. Examples of T_n. Please draw trapezoids for $n = 2$.

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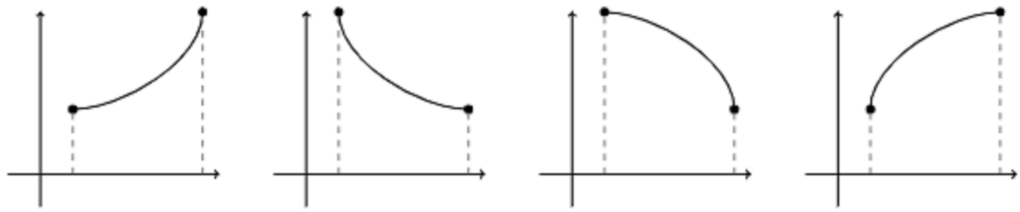
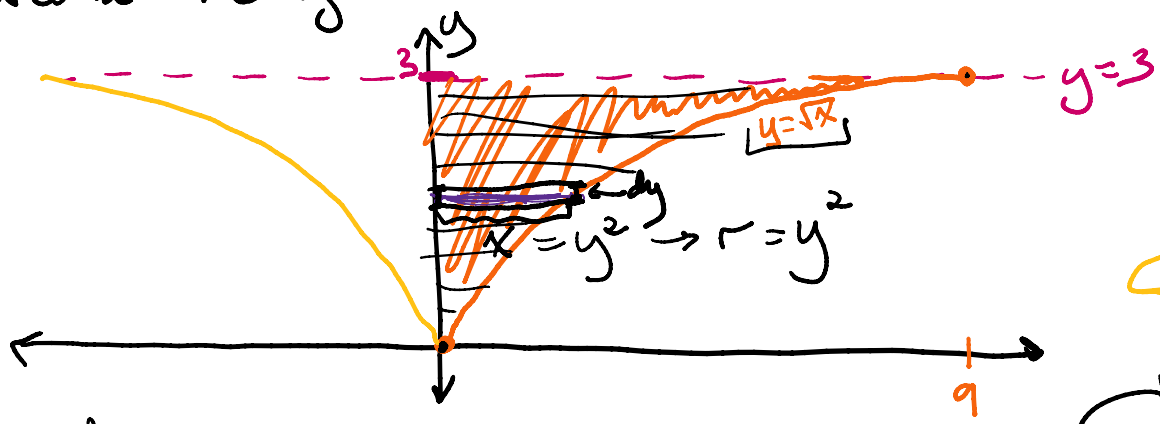


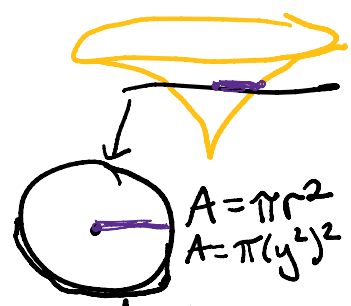
When $f(x)$ is _____, T_n is an overestimate.

When $f(x)$ is _____, T_n is an underestimate.

Rotate the region bounded by $y=\sqrt{x}$, $y=3$ and the y-axis around the y-axis. Find the volume of the solid.



intersects w/ $y=\sqrt{x}$ when
$3=\sqrt{x}$
$9=x$



$A=\pi r^2$
$A=\pi(y^2)^2$

inf. cyl. volume $= A \cdot dy$

$$\int_0^3 \pi y^4\,dy = \ldots$$

$$\int 4x\cos(2-3x)\,dx \qquad \int u\,dv = uv - \int v\,du$$

let $u = 4x$ $\qquad dv = \cos(2-3x)\,dx$

$du = 4dx$ $\qquad v = \dfrac{\sin(2-3x)}{-3}$

$$= \frac{-4x\sin(2-3x)}{3} + \int \frac{\sin(2-3x)}{3}\,4dx$$

$$= \frac{-4x\sin(2-3x)}{3} + \frac{4}{3}\int \sin(2-3x)\,dx$$

$$= \quad '' \quad '' + \frac{4}{3}\left(\frac{+\cos(2-3x)}{+3}\right) + C$$

$$\boxed{\frac{-4x\sin(2-3x)}{3} + \frac{4}{9}\cos(2-3x) + C}$$