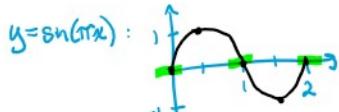


Exam 2

Wednesday, March 9, 2022 10:30 AM

$$a_n = \frac{\sin(\pi n)}{1 + \frac{1}{n}} \quad \text{so} \quad \lim_{n \rightarrow \infty} a_n = 0$$



$$b_n = \frac{\sqrt{n}}{\ln(n)} \quad \lim_{n \rightarrow \infty} \frac{\sqrt{n}}{\ln(n)} \xrightarrow[f(x)=\frac{\sqrt{x}}{\ln(x)}]{\text{use l'Hopital's}} \lim_{x \rightarrow \infty} \frac{\frac{1}{2}x^{-\frac{1}{2}}}{\frac{1}{x}} = \lim_{x \rightarrow \infty} \frac{\frac{1}{2}x^{-\frac{1}{2}}}{\frac{1}{x}} = \lim_{x \rightarrow \infty} \frac{\frac{1}{2}\frac{1}{2}\sqrt{x}}{2} = \infty$$

Diverges to ∞

$$c_n = \frac{(-3)^n}{n!} = \frac{(-3)^1}{1} + \frac{(-3)^2}{2 \cdot 1} + \frac{(-3)^3}{3 \cdot 2 \cdot 1} + \dots + \frac{(-3)^9}{9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} + \dots$$



factorial grows faster than exp $\lim_{n \rightarrow \infty} c_n = 0$

Suppose $\sum_{k=1}^{\infty} a_k$ is a series with the partial sums given by $s_n = \frac{2n^2+1}{n^2+n-1}$.

$$\text{Find } \sum_{k=1}^{\infty} a_k. \quad \sum_{k=1}^n a_k = \frac{2n^2+1}{n^2+n-1}$$

$$\lim_{n \rightarrow \infty} \sum_{k=1}^n a_k = \lim_{n \rightarrow \infty} \left(\frac{2n^2+1}{n^2+n-1} \right) = \frac{2}{1} = 2$$

(i) (4 points) If the sequence $\{a_k\}_{k=1}^{\infty}$ converges, then the sequence $\{s_n\}_{n=1}^{\infty}$ must converge.

Give a convergent sequence $a_k = 1$

Explain why s_n diverges:

as $n \rightarrow \infty$, we are adding up ∞ -ly many 1's, so $\lim_{n \rightarrow \infty} s_n = \infty$

$$\begin{cases} a_n = 1 \\ S_n = \sum_{k=1}^n 1 = 1+1+\dots+1 \\ \text{sequence: } 1, 1, 1, 1 \\ \lim_{n \rightarrow \infty} (1) = 1 \\ S_n = n \\ \lim_{n \rightarrow \infty} S_n = \infty \end{cases}$$

(ii) (4 points) If $a_k \geq a_{k+1}$ for all k , then $s_n \geq s_{n+1}$ for all n .

non-inc. seq. $a_k = 1$

such that $a_k \geq a_{k+1}$ for all k .

Explain why $s_n \geq s_{n+1}$ for all n is not true:

$$S_n = \sum_{k=1}^n 1 = n \quad \text{and} \quad S_{n+1} = \sum_{k=1}^{n+1} 1 = n+1, \text{ so } S_{n+1} > S_n.$$

(iii) (4 points) If $\lim_{n \rightarrow \infty} s_n \neq 0$, then the sequence $\{a_k\}_{k=1}^{\infty}$ must diverge.

Give a sequence $a_k = (\frac{1}{2})^{k-1}$

such that $\lim_{n \rightarrow \infty} s_n \neq 0$.

Explain why a_k converges:

$$\lim_{k \rightarrow \infty} \left(\frac{1}{2} \right)^{k-1} = 0$$

$$\lim_{n \rightarrow \infty} S_n = \sum_{k=1}^{\infty} \left(\frac{1}{2} \right)^{k-1}$$

convergent geo. series w/ $r = \frac{1}{2}$, $a = 1$
($|r| < 1$)

$$= \frac{1}{1-\frac{1}{2}} = 2 \neq 0$$

$$\lim_{k \rightarrow \infty} \frac{1}{2^{k-1}} = 0 \quad \Rightarrow \quad \lim_{k \rightarrow \infty} S_k = \lim_{k \rightarrow \infty} 2^{k-1} = \frac{1}{2}$$

True Statement: If $\lim_{n \rightarrow \infty} a_n \neq 0$, then $\lim_{n \rightarrow \infty} \sum_{k=1}^n a_k$ must diverge.