Wednesday: Parametric Equations

Saturday, February 12, 2022 8:40 PM

Section1.7

Math 2300: Calculus **Spring 2022**

Section 1.7

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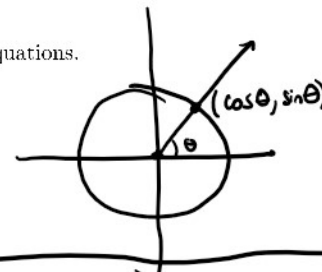
Parametric Equations

A **parametric equation** is one in which the coordinates are given by independent equations.

We have seen one example of this already in the unit circle.



Cartesian Equation: $x^2 + y^2 = 1$

Parametric Equation: $x(\theta) = \cos(\theta)$, $y(\theta) = \sin(\theta)$, $0 \le \theta < 2\pi$ $\quad (\cos(\theta), \sin(\theta)),\ 0 \le \theta < 2\pi$

We call θ the **parameter.** Often, we use t to denote the parameter. $x(t) = -t+2t,\ y(t) = 3t,\ 0 \le t \le 1$

In order to do Calculus, there are three things we want to be able to do with parametric equations:

- Convert from parametric equations to a cartesian equation (regular "$y =$" style)
- Sketch the curves given by parametric equations.
- Write a parametric equation that describes a certain path.

Converting to Cartesian

The process for this is solving each of the parametric equations for t and then setting them equal to each other.

Example 1

Convert the following to a Cartesian equation.

$$x(t) = 2t + 2 \rightarrow x = 2t+2,\quad x-2 = 2t,\quad \frac{x-2}{2} = t$$

$$y(t) = t - 1 \rightarrow y+1 = t$$

$$y+1 = \frac{x-2}{2}$$

$$y = \frac{x}{2} - 1 - 1 \rightarrow y = \frac{x}{2} - 2$$

Example 2

$$x(t) = t^2 + t,\ y(t) = 2t - 1 \rightarrow y + 1 = 2t$$

$$\left(\frac{y+1}{2}\right) = t$$

$$x = \left(\frac{y+1}{2}\right)^2 + \frac{y+1}{2}$$

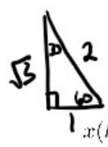
$$x = \frac{1}{4}(y^2 + 2y + 1) + \frac{1}{2}(y+1)$$

$$x = \frac{1}{4}y^2 + \frac{1}{2}y + \frac{1}{4} + \frac{1}{2}y + \frac{1}{2}$$

$$\boxed{x = \frac{1}{4}y^2 + y + \frac{3}{4}}$$

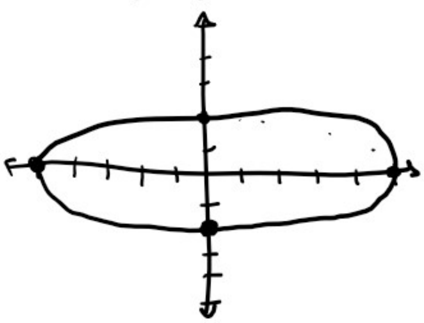
Sketching

Consider the parametric equation



$$x(t) = 5\cos(t)$$
$$y(t) = 2\sin(t)$$

What do you expect it to look like?

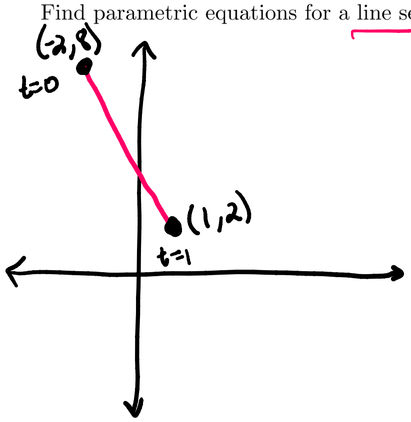


t	$x(t) = 5\cos(t)$	$y(t) = 2\sin(t)$
0	5	0
$\pi/6$	$\frac{5\sqrt{3}}{2}$	1
$\pi/4$	$\frac{5\sqrt{2}}{2}$	$\sqrt{2}$
$\pi/3$	$\frac{5}{2}$	$\sqrt{3}$
$\pi/2$	0	2
π	-5	0
$3\pi/2$	0	-2
2π	5	0

Writing a parametric equation

Example

Find parametric equations for a line segment from $(-2, 8)$ to $(1, 2)$.



let $t \in [0,1]$

$x(t) = mt + b$ $\quad y(t) = mt + b$

$x(0) = -2 \to m(0)+b = -2$, $b = -2$

$x(1) = 1 \to m(1) - 2 = 1$, $m = 3$

$y(0) = 8 \to b = 8$

$y(1) = 2 \to m(1) + b = 2$, $m + 8 = 2$, $m = -6$

$x(t) = 3t - 2$ $\quad y(t) = -6t + 8$ for t in $[0,1]$

Example

Find parametric equations for the path of a particle that moves along the circle $x^2 + (y-1)^2 = 4$, once around clockwise, starting at $(2, 1)$.