Friday Feb 11th

Monday, February 7, 2022 6:40 PM

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(6)

Math 2300: Calculus **Spring 2022**

6.6: Work, Center of Mass

Lecturer: Sarah Arpin

No hydrostatic pressure from this section.

Work = N·m or J Force $[kg \cdot m/s^2 = N]$ Dist = meters

Work = ft·lbs Force = weights lbs. Dist = ft

WORK

Work = Force × Distance (m or ft.)

Weight is force. Force is mass times gravity.

1. A box is slid 3 meters across a carpet against a force of kinetic friction of 45N. How much work is done?

Work = Force × Dist = Area.

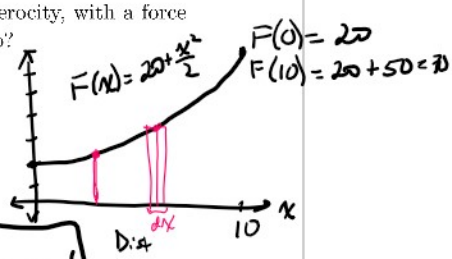
$W = \text{Force} \times \text{Dist.}$

$= 45 \times 3 = \boxed{135 \text{ N·m}}$

2. I am pushing my sister across a 10 foot room. She pushes back with increasing ferocity, with a force of $20 + \frac{x^2}{2}$ pounds, where x is how far I have pushed her. How much work do I do?

After 1ft. of dist: $20 + \frac{1}{2}$ lbs

After 1.1 ft. of dis: $20 + \frac{(1.1)^2}{2}$ lbs

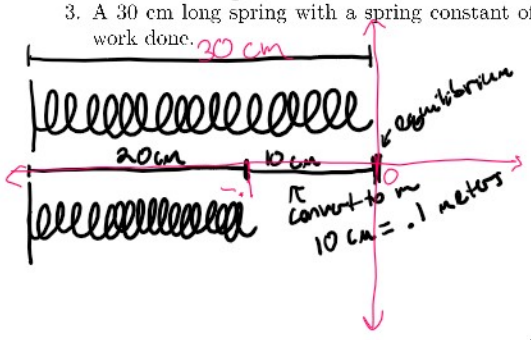


$\text{Force} = 20 + \frac{x^2}{2}$

dist = dx

Need x to go from 0 to 10 and add it up: $\boxed{\int_0^{10} (20 + \frac{x^2}{2})\,dx = W}$

3. A 30 cm long spring with a spring constant of $k = 120$ N/m is compressed to 20 cm. Calculate the work done.



$\boxed{\text{Force} = k \cdot x}$

x = dist from equilibrium, meters

k = spring constant, depends on spring

Force = 120x N

Dist. = dx

$\int_0^{-.1} 120x\,dx = 60x^2 \Big|_0^{-.1} = 60(-.1)^2 = \boxed{.6 \text{ N·m}}$ or $\boxed{.6 \text{ J}}$

.5 ft.

4. A force of 10 lbs is required to hold a spring stretched to 6 inches past its natural length. Calculate the work required to stretch it 8 inches past its natural length.

.5 ft.

Dist $\frac{8 \text{ inches}}{1} \cdot \frac{1 \text{ ft}}{12 \text{ inches}} = \frac{8}{12} \text{ ft} = \frac{2}{3} \text{ ft}$

$F = (20 \text{ lbs/ft}) x \rightarrow \text{lbs}$

$F = kx$

$10 \text{ lbs} = k \cdot 6 \text{ in.}$

$\frac{10 \text{ lbs.}}{6 \text{ in.}} = k$

$F = kx$

$10 \text{ lbs} = k(.5 \text{ ft})$

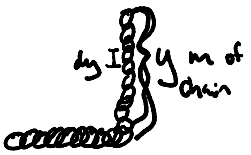
$20 \text{ lbs/ft} = k$

$$W = \int_0^{2/3} 20x \, dx = 10x^2 \Big|_0^{2/3} = \frac{10 \cdot 4}{9} = \boxed{\frac{40}{9} \text{ ft} \cdot \text{lbs}}$$

~~5. How much energy is required to hoist a 3 kg pumpkin 15 meters to the roof of the math building?~~

~~6. How much energy is required to carry a 44-lb stack of books up to the third floor of the math building?~~

7. A 6-kg chain is 3 meters long. How much work is done lifting it from the ground until its lower end is 2 meters off of the ground?



Part I: get 3m chain entirely off ground
Part II: lift whole thing extra 2m.

* How many kg is y meters of chain?

6 kg for all 3m chain

$\frac{6\text{ kg}}{3\text{ m}} = \frac{?}{y\text{ m}} \rightarrow$ $2y$ kg.

Part II: We are lifting
How many kg? 6 kg
How much force? (6)(9.8) kg m/s²

How much force does it take to lift $2y$ kg?

mult. by gravity: $g = 9.8\ m/s^2$

$F = (2y\ kg)(9.8\ m/s^2)$ ← units $kg \cdot m/s^2 = N$

Part I: $\int_0^3 (2y)(9.8)\,dy = 9.8y^2\Big|_0^3 = 9\cdot(9.8)\ J = 88.2\ J$

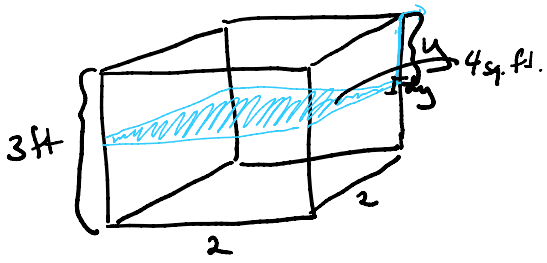
Part II: $\int_0^2 (6)(9.8)\,dy = \int_3^5 (6)(9.8)\,dy = (6)(9.8)(2) = 117.6\ J$

$88.2 + 117.6 = 205.8\ J$

Force × dist = (6)(9.8)(2)

8. How much work is done by emptying a $2 \times 2 \times 3$-foot rectangular tank? The water must be pumped to a point in the upper corner of the tank.

Density of water: 1000 kg/m³ or 62.4 lbs/ft³



Force = Density × Volume

$= (62.4)(4\,dy)$

$W = \int_0^3 (62.4)(4)\,dy$

9. A tub has the shape of the solid of revolution formed by rotating around the y-axis the portion of the curve $y = 2x^4$ that lies between $x = 0$ and $x = 1$. (Draw it.) How much work is done to empty the tank? The water must be pumped out of the top of the tank.

Moments and Center of Mass

1. Find the moments M_x and M_y and the center of mass of the system of the following point masses:
   - A mass of 6 at the point $(1, 5)$,
   - a mass of 5 at the point $(3, -2)$,
   - a mass of 10 at the point $(-2, -1)$.

2. Find the centroid of the region bounded by the curves $y = \sqrt{x}$ and $y = x$.

3. Find the center of mass of the semicircular plate of radius r.

4. Find the center of mass of the region between the x-axis and the parabola $y = x^2 + 1$ between $x = -2$ and $x = 2$.