Wednesday Feb 9th

Monday, February 7, 2022 6:39 PM

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Math 2300: Calculus **Spring 2022**

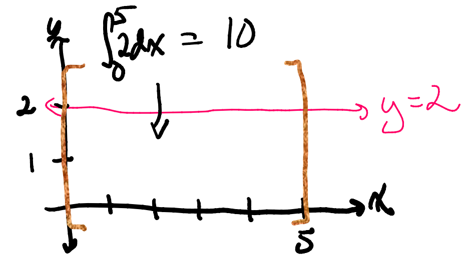
6.5: Average Value of a Function

Lecturer: Sarah Arpin

Average Value Formula

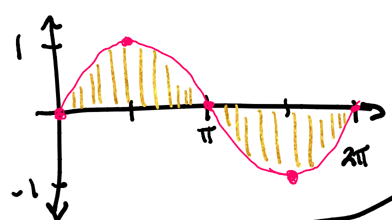
What is the average value of the function $y = 2$ on the interval $[0, 5]$?

$\int_0^5 2\,dx = 10$



Average value $= 2$

What is the average value of the function $y = \sin(x)$ on the interval $[0, 2\pi]$?



The avg value $= \dfrac{f_1 + \cdots + f_n}{n}$

We have ∞-ly many values

$\boxed{\text{The average value here is } 0.}$

What is the average value of $f(x)$ on $[a,b]$?

$$\frac{1}{b-a}\int_a^b f(x)\,dx$$

$$\boxed{f_{\text{ave}} := \frac{1}{b-a}\int_a^b f(x)\,dx}$$

Example

Find the average value of the function $f(x) = \ln(x)$ on the inerval $[1, 3]$.

$\int u\,dv = uv - \int v\,du$

$u = \ln(x) \quad du = \frac{1}{x}dx$

$dv = dx \quad v = x$

$$f_{\text{ave}} = \frac{1}{3-1}\int_1^3 \ln(x)\,dx = \frac{1}{2}\int_1^3 \ln(x)\,dx$$

$$= \frac{1}{2}\left[x\ln(x)\Big|_1^3 - \int_1^3 1\,dx\right]$$

$$= \frac{1}{2}\left[3\ln(3) - \ln(1) - \left[x\Big|_1^3\right]\right] = \frac{1}{2}\left[3\ln(3) - (2)\right] = \boxed{\frac{3}{2}\ln(3) - 1}$$

Mean Value Theorem (Calc I): If f cont. on $[a,b]$, then $\exists$ c in $[a,b]$ where

$$f'(c) = \frac{f(b)-f(a)}{b-a}$$

Mean Value Theorem for Integrals

If f is continuous on $[a,b]$, then there exists a number c in $[a,b]$ such that $f(c) = f_{\text{ave}}$. That is:

$$\boxed{\int_a^b f(x)dx = f(c)(b-a).}$$

$$f(c) = \frac{1}{b-a}\int_a^b f(x)dx$$

Proof

Prove the Mean Value Theorem for Integrals by applying the Mean Value Theorem for derivatives to the function $F(x) = \int_0^x f(t)dt$.

Want to show that on $[a,b]$, there exists a c w/

$$f(c) = \frac{1}{b-a}\int_a^b f(x)dx$$

If $F(x)$ is cont. on $[a,b]$, then by Calc I MVT, there exists c in $[a,b]$ w/

$$F'(c) = \frac{F(b)-F(a)}{b-a}.$$

$$\underbrace{\frac{d}{dx}\left(\int_0^x f(t)dt\right)}_{f(x)\big|_{x=c}}\Bigg|_{x=c} = \frac{\int_0^b f(t)dt - \int_0^a f(t)dt}{b-a}$$

$$= \frac{1}{b-a}\left(\int_0^b f(t)dt + \int_a^0 f(t)dt\right)$$

$$\star \quad \boxed{f(c) = \frac{1}{b-a}\int_a^b f(t)dt}$$