Tuesday Feb 8th

Monday, February 7, 2022 6:39 PM

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Math 2300: Calculus II **Spring 2022**

Section 6.3

Lecturer: Sarah Arpin

We've been rotating to obtain solids with nice cross sections: disk or washer shapes. What if our cross sections are not disks or washers?

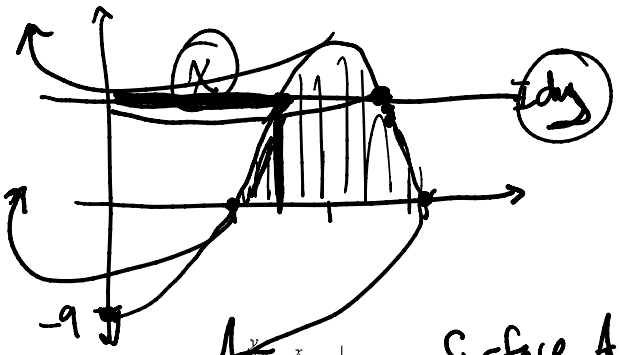
Volumes of Solids of Revolution by Cylindrical Shells

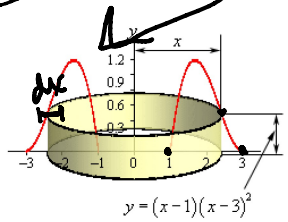
Motivating Example

Find the volume obtained by rotating the region bounded by $y = (x-1)(x-3)^2$ and the x-axis about the y-axis.

can't solve for x! can't find the radius

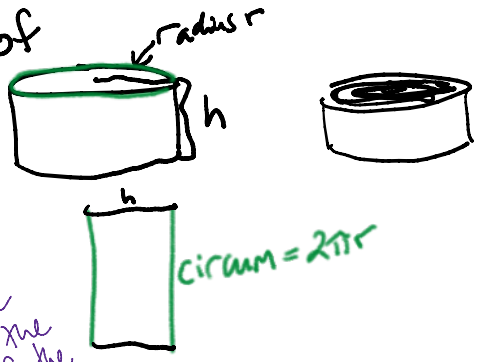
$y(0) = (-1)(-3)^2 = -9$





Surface Area of a cylinder: (just around side, not top + bottom)

$SA = 2\pi r h$



$$V = \int_1^3 2\pi \overbrace{(x)}^{r} \overbrace{(x-1)(x-3)^2}^{h} \, dx$$

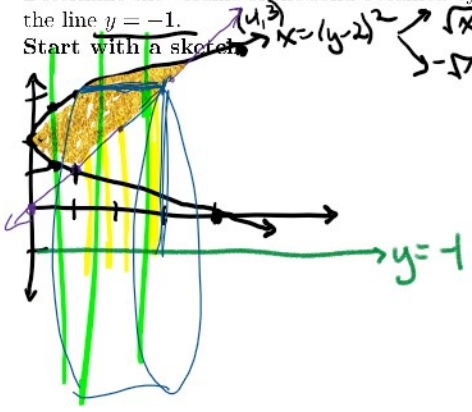
direction we thicken the walls of the cylinder

x	y
0	2
1	1
4	0

Example 2

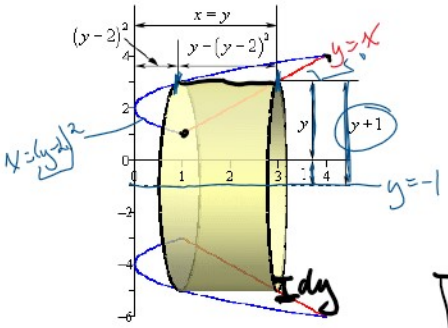
Determine the volume of the solid obtained by rotating the region bounded by $x = (y-2)^2$ and $y = x$ about the line $y = -1$.

Start with a sketch:



intersection of $y=x$ and $x=(y-2)^2$

x



$SA = 2\pi rh$

$h = (y=x \text{ line } x\text{-value}) - (x=(y-2)^2 \ x\text{-value})$

$h = y - (y-2)^2$

$r = y+1$

$SA = 2\pi(y+1)(y-(y-2)^2)$

$$V = \int_1^4 2\pi(y+1)(y-(y-2)^2)\,dy$$

$y=x$ int. $(y-2)^2=x$

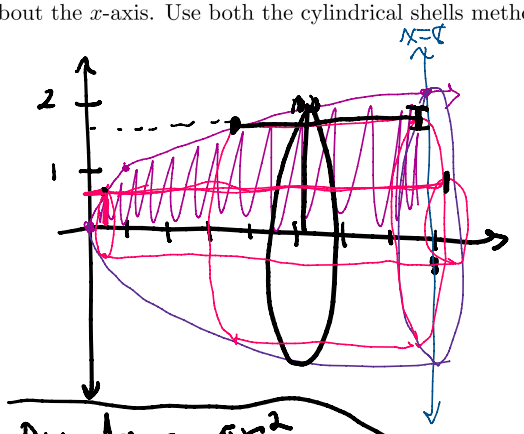
$y = (y-2)^2 \rightarrow y = y^2-4y+4 \rightarrow 0 = y^2-5y+4$

$0 = (y-4)(y-1)$

$y=4,\ y=1$

Example 3:

Determine the volume of the solid obtained by rotating the region bounded by $y = \sqrt[3]{x}, x = 8$ and the x-axis about the x-axis. Use both the cylindrical shells method and the disk/washer method!



$y = \sqrt[3]{x}$

$y^3 = x$

x	y
0	0
1	1
8	2

Cylinder Method

$SA = 2\pi rh$ $\quad dy$

want r, h in terms of y.

$r = y$

$h = 8 - y^3$

$$\int_0^2 2\pi y(8-y^3)\,dy = 2\pi \int_0^2 (8y - y^4)\,dy$$

$$= 2\pi \left(4y^2 - \frac{y^5}{5} \Big|_0^2 \right)$$

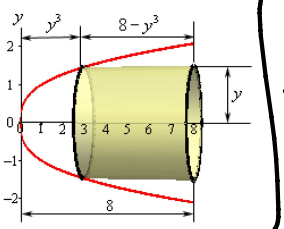
$$= 2\pi \left(16 - \frac{32}{5}\right) = 2\pi(16 - 6.4)$$

$$= 2\pi \cdot 9.6$$

$$= \boxed{19.2\pi}$$

Disk Area $= \pi r^2$

$= \pi \left(\sqrt[3]{x}\right)^2$



$$\int_0^8 \pi x^{2/3}\,dx$$

$$= \pi \frac{x^{5/3}}{5/3} \Big|_0^8$$

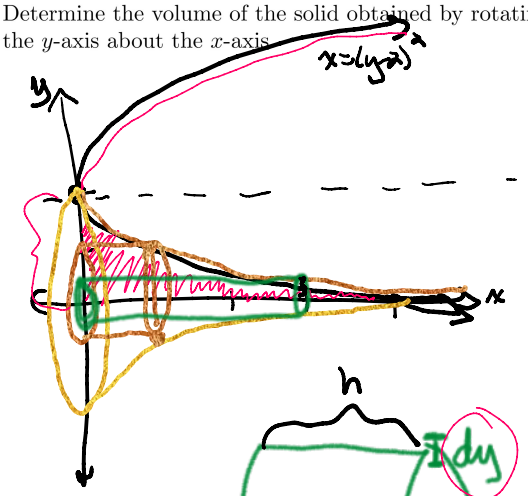
$$= \pi \frac{8^{5/3}}{5/3}$$

$$= \pi \frac{32 \cdot 3}{5}$$

$$= \pi \cdot \frac{96}{5} = \boxed{19.2\pi}$$

Example 4:

Determine the volume of the solid obtained by rotating the region bounded by $x = (y-2)^2$, the x-axis and the y-axis about the x-axis.



x-int: set $y = 0$:

$x = 4$

DISK METHOD

dx

y

Need to solve for y in terms of x

$x = (y-2)^2$

$\pm\sqrt{x} = y - 2$

$2 \pm \sqrt{x} = y$

want: $2 - \sqrt{x}$

$r = 2 - \sqrt{x}$

$A = \pi r^2$

$= \pi(2-\sqrt{x})^2$

$V = \int_0^4 \pi(2-\sqrt{x})^2 dx$

$SA = 2\pi rh$

$r = y$

$h = x = (y-2)^2$

$$V = \int_0^2 2\pi y(y-2)^2 dy$$