6.1, 6.2

Thursday, February 3, 2022 2:49 PM

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Math 2300: Calculus **Spring 2022**

Lecture 15: Friday February 4

Lecturer: Sarah Arpin

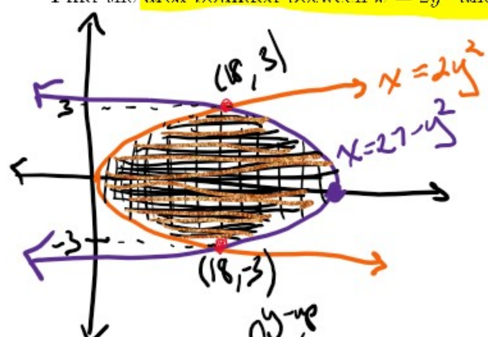
§6.1 and 6.2: The last two sections on the exam on Monday! WebAssign due dates have been moved - check WebAssign.

15.1 Areas of Regions and Volumes of Solids of Revolution

The goal of today is to review ways of finding geometric areas, and to extend that notion to volumes. **Always draw a picture first.**

15.1.1 Example 1:

Find the area bounded between $x = 2y^2$ and $x = 27 - y^2$.



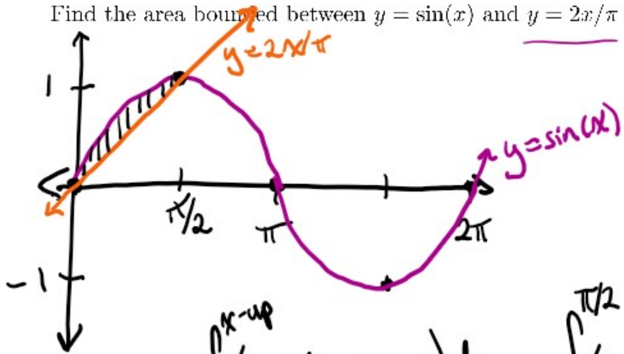
Find intersection:

$$2y^2 = 27 - y^2$$
$$3y^2 = 27$$
$$y^2 = 9$$
$$y = \pm 3$$
$$x = 2(3)^2 = 18$$

$$\int_{y\text{-low}}^{y\text{-up}} (\text{purple} - \text{orange})\, dy = \int_{-3}^{3} (27 - y^2 - 2y^2)\, dy$$
$$= \int_{-3}^{3} (27 - 3y^2)\, dy$$
$$= 27y - y^3 \Big|_{-3}^{3}$$
$$= 27(3) - 27 - (27(-3) + 27)$$
$$= 27(2) + 27(2) = \boxed{108}$$

15.1.2 Example 2:

Find the area bounded between $y = \sin(x)$ and $y = 2x/\pi$ for $x > 0$.



When is $y = 2x/\pi$ at height 1?

$$1 = 2x/\pi$$

$$\frac{\pi}{2} = x$$

$$\int_{x\text{-low}}^{x\text{-up}} (\text{purple} - \text{orange})\,dx = \int_0^{\pi/2} \left(\sin x - \frac{2x}{\pi}\right) dx = -\cos(x) - \frac{x^2}{\pi} \Big|_0^{\pi/2}$$

$$= \left(0 - \frac{(\pi/2)^2}{\pi}\right) - (-1 - 0)$$

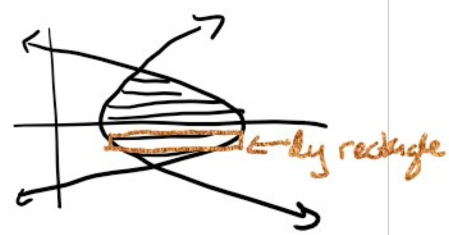
$$= \boxed{-\frac{\pi}{4} + 1}$$

15.1.3 Summary

It's either

$$\int_{x=a}^{x=b} (\text{top} - \text{bottom})\,dx \quad \text{or} \quad \int_{y=a}^{y=b} (\text{right} - \text{left})\,dy.$$

shade vertically



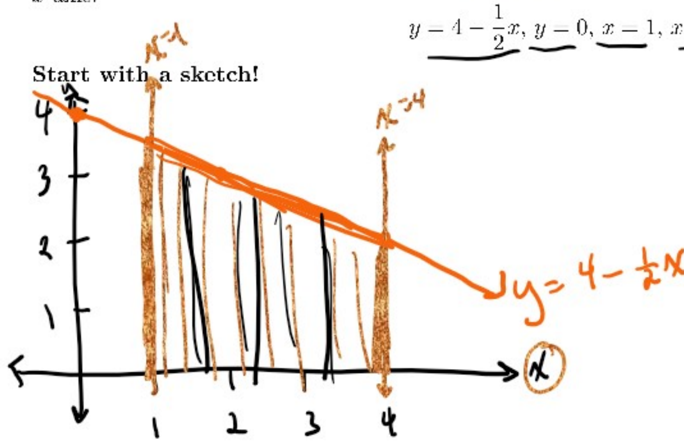
15.2 Volumes

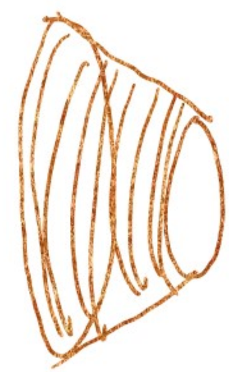
15.2.1 Example:

Find the volume of the solid V obtained by rotating the region bounded by the following curves around the x-axis:

$$y = 4 - \frac{1}{2}x,\ y = 0,\ x = 1,\ x = 4$$

Start with a sketch!





Vertical cross sections
are circles! $A = \pi r^2$
Thicken in x-direction → dx integral.

$$\int_1^4 \pi \left(4 - \tfrac{1}{2}x\right)^2 dx = \pi \int_1^4 \left(16 - 4x + \tfrac{1}{4}x^2\right) dx$$

$$= \pi \left(16x - 2x^2 + \tfrac{1}{12}x^3\right) \Big|_1^4$$

$$= \pi \left[\left(\underbrace{64 - 32}_{32} + \tfrac{64}{12}\right) - \left(\underbrace{16 - 2}_{14} + \tfrac{1}{12}\right)\right]$$

$$= \pi \left[18 + \tfrac{63}{12}\right] = \left(\tfrac{21}{4} + \tfrac{72}{4}\right)\pi$$

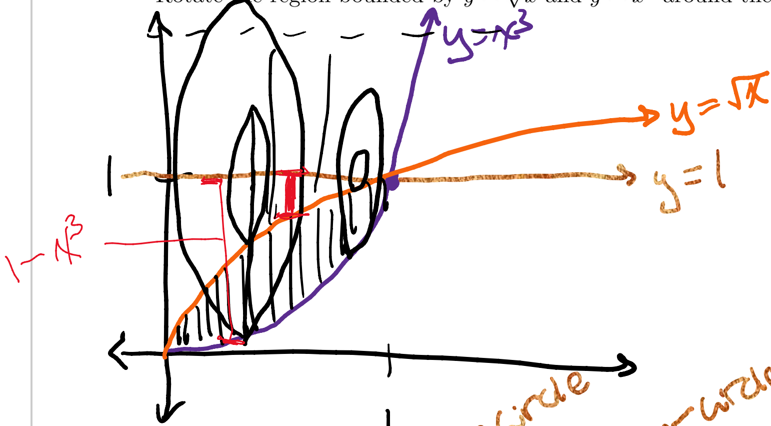
$$= \frac{93\pi}{4}$$

This is what we will call **washer method**.

15.2.2 Another Example:

intersect at $x=0,1$

Rotate the region bounded by $y = \sqrt{x}$ and $y = x^3$ around the line $y = 1$.



Vert. cross sections look like:

express these areas

outer circle area

inner circle area

Thicken in x-direction w/ dx,

$$\pi r_1^2 - \pi r_2^2 = \pi\left(r_1^2 - r_2^2\right)dx$$

Outer radius = distance from purple curve to line $y=1$

inner radius = dist. from orange curve to line $y=1$

$$\pi\int_0^1\left((1-x^3)^2 - (1-\sqrt{x})^2\right)dx = \pi\int_0^1\left((\cancel{1}-2x^3+x^6)-(\cancel{1}-2\sqrt{x}+x)\right)dx$$

$$=\pi\int_0^1\left(x^6-2x^3-x+2x^{1/2}\right)dx$$

$$=\pi\left(\frac{x^7}{7}-\frac{x^4}{2}-\frac{x^2}{2}+\frac{2x^{3/2}}{3/2}\right)\Bigg|_0^1$$

$$=\pi\left(\frac{1}{7}\underbrace{-\frac{1}{2}-\frac{1}{2}+\frac{4}{3}}\right)$$

$$=\pi\left(\frac{1}{7}+\frac{1}{3}\right) = \boxed{\frac{10\pi}{21}}$$