



Day11

Math 2300: Calculus	Spring 2019
Lecture 9: Friday January 25	
Lecturer: Sarah Arpin	Scribes:

~~WebAssign due tonight~~

9.1 Partial Fractions

9.1.1 Last ^{Week} Class...

Last class we learned out to do trig substitutions. In summary,

If you need to get rid of...	Use...
$\sqrt{a^2 + x^2}$	$x = a \tan(\theta)$
$\sqrt{a^2 - x^2}$	$x = a \sin(\theta)$
$\sqrt{x^2 - a^2}$	$x = a \sec(\theta)$

9.1.2 Warmup

Evaluate the following integrals:

$\int_0^1 \frac{1}{x+1} dx = \ln|x+1| + C$
 $\int \frac{1}{x+1} dx = \ln|x+1| + C$
 $u = x+1$
 $du = dx$
 $\frac{1}{2} du = dx$
 $\frac{1}{2} \ln|2x+1| \Big|_0^1$
 $\frac{1}{2} \ln(3) - \frac{1}{2} \ln(1)$
 $\frac{1}{2} \ln(3)$

$\int \frac{1}{(x-7)^2} dx = \frac{1}{-1} x + C$
 $\int x^{-2} dx = \frac{x^{-1}}{-1} + C$
 $= \frac{-1}{(x-7)} + C$

Yesterday you got an introduction to the method of partial fractions. This can be useful when integrating rational functions that don't have a straightforward anti-derivative. There are three types that we need to be concerned with, and mixtures are allowed. Also, note that in order for these formulas to work, the

Linear Factors:	$\frac{ax^2 + bx + c}{(x+d)(x+e)} = \frac{A}{x+d} + \frac{B}{x+e}$
Repeated Linear Factors:	$\frac{ax^2 + bx + c}{(x+d)^2} = \frac{A}{x+d} + \frac{B}{(x+d)^2}$
Irreducible Quadratic Factors:	$\frac{ax^2 + bx + c}{(x^2 + px + q)(x+r)} = \frac{Ax + B}{x^2 + px + q} + \frac{C}{x+r}$

degree of the numerator must be less than the degree of the denominator. You might need to do polynomial long division in order to insure that this happens. Let's do some examples of each type:

9.1.3 Example 1:

$$\int \frac{4}{x^2 + 5x - 14} dx = \int \frac{4}{(x+7)(x-2)} dx = \int \frac{-4/9}{x+7} dx + \int \frac{4/9}{x-2} dx$$

$$\frac{4}{(x+7)(x-2)} = \left(\frac{A}{x+7} + \frac{B}{x-2} \right) \frac{1}{(x+7)(x-2)} = \frac{-4}{9} \int \frac{1}{x+7} dx + \frac{4}{9} \int \frac{1}{x-2} dx$$

$$4 = A(x-2) + B(x+7)$$

$$4 = Ax - 2A + Bx + 7B$$

$$\underline{0x + 4} = x(A+B) - 2A + 7B$$

$$\begin{aligned} 0 &= A+B & 4 &= -2A+7B \\ -B &= A & 4 &= -2(-B)+7B \\ & & 4 &= 9B \\ & & \frac{4}{9} &= B, A = -\frac{4}{9} \end{aligned}$$

$$= \frac{-4}{9} \ln|x+7| + \frac{4}{9} \ln|x-2| + C$$

9.1.4 Example 2

$$\begin{array}{r}
 x^3 \\
 x^2 + 2x + 1 \overline{) x^3 + 0x^2 + 0x + 0} \\
 \underline{-(x^3 + 2x^2 + x)} \\
 -2x^2 - x + 0 \\
 \underline{-(-2x^2 - 4x - 2)} \\
 3x + 2
 \end{array}$$

"what do you mult.
 x^2 by to get x^3 ?"

"what do you mult.
 x^2 by to get $-2x^2$?"

← deg 1 < deg 2, so this
 is remainder

$$\frac{x^3}{x^2 + 2x + 1} = (x-2) + \frac{3x+2}{x^2+2x+1}$$

$$\frac{3x+2}{x^2+2x+1} = \frac{3x+2}{(x+1)^2} = \frac{A}{x+1} + \frac{B}{(x+1)^2} \quad \text{clear denoms.}$$

$$3x+2 = A(x+1) + B$$

$$3x+2 = Ax + A+B$$

$$3 = A \quad A+B = 2$$

$$B = -1$$

$$= \int (x-2) dx + \int \frac{3x+2}{x^2+2x+1} dx$$

$$= \int (x-2) dx + \int \frac{3}{x+1} dx + \int \frac{-1}{(x+1)^2} dx$$

$$= \int (x-2) dx + 3 \int \frac{1}{x+1} dx - \int \frac{1}{(x+1)^2} dx$$

$$= \frac{x^2}{2} - 2x + 3 \ln|x+1| + (x+1)^{-1} + C$$

9.1.5 Example 3:

$$\int \frac{10}{(x+1)(x^2+9)} dx = \int \frac{1}{x+1} dx + \int \frac{-x+1}{x^2+9} dx$$

$$\frac{10}{(x+1)(x^2+9)} = \frac{A}{x+1} + \frac{Bx+C}{x^2+9} = \int \frac{1}{x+1} dx - \int \frac{x}{x^2+9} dx + \int \frac{1}{x^2+9} dx$$

$\underbrace{\int \frac{1}{x+1} dx}_{\ln}$
 $\underbrace{\int \frac{x}{x^2+9} dx}_{u\text{-sub}}$
 $\underbrace{\int \frac{1}{x^2+9} dx}_{\sin: \frac{1}{x^2+1}}$

$$10 = A(x^2+9) + (Bx+C)(x+1)$$

$$10 = Ax^2 + 9A + Bx^2 + Bx + Cx + C$$

$$10 = x^2(A+B) + x(B+C) + 9A+C$$

$$A+B=0 \quad B+C=0 \quad 9A+C=10$$

$$A=-B \quad C=-B \quad 9(-B)+-B=10$$

$$\underline{A=1} \quad \underline{C=1} \quad -10B=10$$

$$\underline{B=-1}$$

$$= \ln|x+1| - \frac{1}{2} \ln|x^2+9| + \frac{1}{3} \arctan\left(\frac{x}{3}\right) + C$$