Day 06

Tuesday, January 18, 2022 10:49 AM

2300_Spri...
(2)

Math 2300: Calculus **Spring 2022**

Day 6

Lecturer: Sarah Arpin

6.1 Warm-up

$$\cos^2 x = \frac{1+\cos(2x)}{2}$$

Evaluate: $\int \cos^2(x)dx$

$$= \int \frac{1+\cos(2x)}{2}\,dx = \int \left(\frac{1}{2} + \frac{1}{2}\cos(2x)\right)dx$$

$$= \frac{1}{2}\int (1+\cos(2x))\,dx$$

$$= \frac{1}{2}\left(x + \frac{1}{2}\sin(2x)\right) + C$$

$$= \boxed{\frac{1}{2}x + \frac{1}{4}\sin(2x) + C}$$

$$\boxed{1+\tan^2(x)=\sec^2(x)}$$

6.2 Products of Powers of Secant and Tangent

$du = \sec^2(x)dx$

- If power on $\sec(x)$ is even: Use $u = \tan(x)$, and put aside $\sec^2(x)dx$ part of integrand. Convert remaining even powers of secant to tangent using the pythagorean identity.

  For example:

$$\int \tan^4(x)\underbrace{\sec^2(x)dx}$$

$u=\tan(x)$
$du=\sec^2(x)dx$

$$\int u^4\,du = \frac{1}{5}u^5 + C$$
$$= \boxed{\frac{1}{5}\tan^5(x)+C}$$

$du=\tan(x)\sec(x)dx$

- If power on $\tan(x)$ is odd and $\sec(x)$ is odd: Use $u = \sec(x)$, put aside $\tan(x)\sec(x)dx$ part of the integral and convert the remaining even powers of tangent to secant using the pythagorean identity.

  For example:

$$\int \tan^3(x)\sec^5(x)dx = \int \tan^2(x)\sec^4(x)\left[\tan(x)\sec(x)dx\right]$$

$u=\sec(x)$ $\quad du=\tan(x)\sec(x)dx$

$1+\tan^2 x = \sec^2 x \longrightarrow \tan^2 x = \boxed{\sec^2 x - 1}$

$$\ldots = \int(\sec^2(x)-1)\sec^4(x)(\tan(x)\sec(x)dx)$$
$$= \int(u^2-1)u^4\,du = \int(u^6-u^4)\,du = \frac{1}{7}u^7 - \frac{1}{5}u^5 + C$$
$$= \frac{1}{7}\sec^7(x) - \frac{1}{5}\sec^5(x) + C$$

- If power on $\sec(x)$ is odd and even power on $\tan(x)$ is even...bleh. It's going to be long and tricky and likely require integration by parts and u-substitution. This is not ideal...

6.3 Choosing a good u, dv

This one is difficult, so we have a difficult example to work with. Try to use integration by parts on:

*good one to memorize $\int \sec(x)dx$ Multiply by $(\sec x + \tan x)$

$$= \int \frac{\sec(x)}{1} \cdot \frac{\sec(x)+\tan(x)}{\sec(x)+\tan(x)}\,dx = \int \frac{\sec^2(x) + \sec(x)\tan(x)}{\sec(x)+\tan(x)}\,dx$$

$$u = \sec(x) + \tan(x)$$

$$du = (\sec(x)\tan(x) + \sec^2(x))dx$$

$$= \int \frac{1}{u}\,du = \ln|u| + C = \boxed{\ln|\sec(x)+\tan(x)| + C}$$

$$\int \sec^3(x)dx = \int \underline{\sec(x)}\,\underline{\sec^2(x)dx}$$

let $u = \sec(x)$ $du = \sec(x)\tan(x)dx$

$dv = \sec^2(x)dx$ $v = \tan(x)$

$\tan^2(x) = \sec^2(x) - 1$

$$\int \sec^3(x)dx = \sec(x)\tan(x) - \int \tan^2(x)\sec(x)dx$$

$$\int \sec^3(x)dx = \sec(x)\tan(x) - \int(\sec^3(x) - \sec(x))dx$$

$$\int \sec^3(x)dx = \sec(x)\tan(x) - \int \sec^3(x)dx + \underline{\int \sec(x)dx}$$

$$2\int \sec^3(x)dx = \sec(x)\tan(x) + \ln|\sec(x)+\tan(x)| + C$$

$$\int \sec^3(x)dx = \frac{1}{2}\sec(x)\tan(x) + \frac{1}{2}\ln|\sec(x)+\tan(x)| + C$$

In summary:

Situation	How to start?
$\int \sin^{\text{odd}}(x)\cos^{n}(x)dx$	Do u-sub with $u = \cos(x)$
$\int \sin^{n}(x)\cos^{\text{odd}}(x)dx$	Do u-sub with $u = \sin(x)$
$\int \sin^{\text{even}}(x)\cos^{\text{even}}(x)dx$	Use power-reducing formulas
$\int \tan^{n}(x)\sec^{\text{even}}(x)dx$	Do u-sub with $u = \tan(x)$
$\int \tan^{\text{odd}}(x)\sec^{\text{odd}}(x)dx$	Do u-sub with $u = \sec(x)$
$\int \tan^{\text{even}}(x)\sec^{\text{odd}}(x)dx$	Going to be complicated...boomerang.

Note: 0 is an even number.