Day 05

Thursday, January 13, 2022 4:47 PM

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Math 2300: Calculus	Spring 2022

Day 5

Lecturer: Sarah Arpin

5.1 Warm-up

$$\int u\,dv = uv - \int v\,du \quad \star$$

$$\int \underset{u}{\sin(x)} \underset{dv}{e^x\,dx}$$

$$\boxed{\begin{array}{ll} u = e^x & du = e^x dx \\ dv = \sin(x)dx & v = -\cos(x) \end{array}}$$

$$= -e^x\cos(x) + \boxed{\int \cos(x)e^x dx}$$

$u = e^x \qquad du = e^x dx$

$dv = \cos(x)dx \qquad v = \sin(x)$

$$\int \sin(x)e^x dx = -e^x\cos(x) + \left[e^x\sin(x) - \int \sin(x)e^x dx\right]$$

$$+\int \sin(x)e^x dx \qquad\qquad +\int \sin(x)e^x dx$$

$$2\int \sin(x)e^x dx = -e^x\cos(x) + e^x\sin(x) + C$$

$$\int \sin(x)e^x dx = \frac{1}{2}\left(-e^x\cos(x) + e^x\sin(x)\right) + C$$

5.2 Trigonometric Integrals

Today we're going to learn three techniques to evaluate integrals involving trig functions.

1. How to use trig identities to help evaluate trig integrals
2. How to choose a good u to use u-substitution in evaluating trig integrals.
3. How to make good u, dv choices to use integration by parts in evaluating trig integrals.

5.2.1 Trig Identities

$$\int_0^1 (\sin^2 x + \cos^2 x)\,dx = \int_0^1 1\,dx = x\Big|_0^1 = 1$$

You should be able to use these easily:

Pythagorean $a^2+b^2=c^2$

1. $\sin^2(x) + \cos^2(x) = 1$
2. $\tan^2(x) + 1 = \sec^2(x)$
3. $1 + \cot^2(x) = \csc^2(x)$

power-reducing

4. $\sin^2(x) = \frac{1-\cos(2x)}{2}$
5. $\cos^2(x) = \frac{1+\cos(2x)}{2}$
6. $\sin(2x) = 2\cos(x)\sin(x)$

$\cos(2x) = 2\cos^2 x - 1$

Double-angle

$\int \cos(2\theta)\,dx$

What $f(\theta)$ has $f'(\theta) = \cos(2\theta)$?

$\boxed{\frac{d}{dx}\sin(x) = \cos(x)}$

$\frac{d}{dx}\sin(2x) = \cos(2x)\cdot 2$

$\frac{d}{dx}\left(\frac{1}{2}\sin(2x)\right) = \cancel{\frac{1}{2}}\cos(2x)\cdot\cancel{2}$

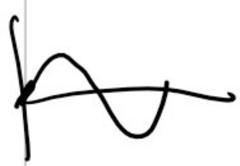
$\int \cos(2\pi x)\,dx = \frac{\sin(2\pi x)}{2\pi} + C$

Example:

$$\int \sin^2(\theta)d\theta = \int \frac{1-\cos(2\theta)}{2}\,d\theta = \frac{1}{2}\int (1-\cos(2\theta))\,d\theta$$

$$= \frac{1}{2}\left(\theta - \frac{1}{2}\sin(2\theta)\right) + C$$

$$= \boxed{\frac{1}{2}\theta - \frac{1}{4}\sin(2\theta) + C}$$



$\sin^2(x)+\cos^2(x)=1$

$\cos^2(x) = 1-\sin^2(x)$

$\cos^4(x) = (1-\sin^2(x))^2$

5.2.2 Choosing a good u

5.2.2.1 Products of Powers of Sine and Cosine

- If at least one power is odd, pull one of these out to put with the dx and convert the remaining even powers to an expression involving the other trig function using pythagorean identities. Let $u =$ the antiderivative of this trig function.

  For example:
  $\int \sin^4(x)\cos^5(x)dx = \int \sin^4(x)\cos^4(x)\,\underline{\cos(x)\,dx}$

  $= \int \sin^4(x)\left(1-\sin^2(x)\right)^2 \underline{\cos(x)dx}$

  $\boxed{\text{let } u=\sin(x) \quad du=\cos(x)dx}$

  $= \int u^4(1-u^2)^2 du = \int u^4(1-2u^2+u^4)du$

  $= \int (u^4-2u^6+u^8)du = \frac{u^5}{5} - \frac{2u^7}{7} + \frac{u^9}{9} + C$

  $= \frac{\sin^5(x)}{5} - \frac{2\sin^7(x)}{7} + \frac{\sin^9(x)}{9} + C$

  **Note, we still could have used this if it was $\int \sin^3(x)\cos^5(x)dx$! We use this method anytime *at least* one of the powers is odd.

- If both powers are even, use the power-reducing identity, maybe twice.

  For example:
  $\int \sin^2(x)\cos^2(x)dx = \int \left(\frac{1-\cos(2x)}{2}\right)\left(\frac{1+\cos(2x)}{2}\right)dx$

  $= \frac{1}{4}\int (1-\cos(2x))(1+\cos(2x))\,dx$

  $= \frac{1}{4}\int (1-\cos^2(2x))dx$

  $= \frac{1}{4}\int \left(1 - \left(\frac{1+\cos(4x)}{2}\right)\right)dx$

  $= \frac{1}{4}\int \left(\frac{1-\cos(4x)}{2}\right)dx$

  $= \frac{1}{8}\int (1-\cos 4x)dx = \frac{1}{8}\left(x - \frac{\sin(4x)}{4}\right) + C$