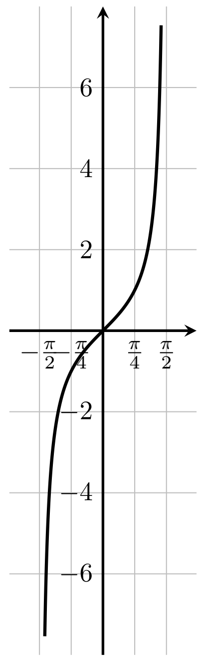
Project: the arctangent function

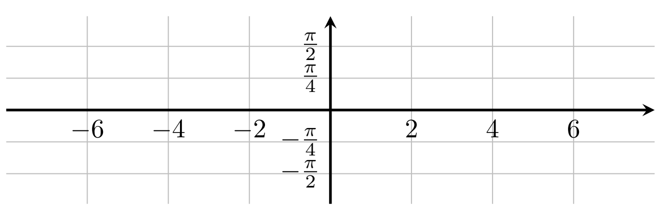
1. On the axes below is a graph of the function $y = f(x) = \tan x$, for $-\frac{\pi}{2} < x < \frac{\pi}{2}$.



(As x approaches $-\pi/2$ from the right, the graph of $f(x)$ shoots off towards $-\infty$; as x approaches $\pi/2$ from the left, the graph of $f(x)$ shoots off towards ∞.)

(a) Explain why this function $f(x)$, when restricted to the given domain, has an inverse function $f^{-1}(x) = \arctan x$. (Pronounced "arctangent of x.")

(b) Sketch the graph of $y = f^{-1}(x) = \arctan x$, on the axes below.

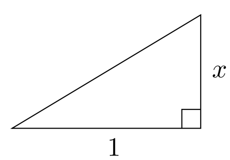


2. Note that, because we have defined $\arctan x$ as the inverse of $\tan x$, we have:

$$\tan(\arctan x) = x.$$

Differentiate both sides of this equation to find a formula for the derivative of $\arctan x$. Express your answer in terms of $\sec^2(\arctan x)$. (You'll need to recall that $\frac{d}{dx}[\tan x] = \sec^2 x$.)

3. Referring to the triangle below, explain why $\sec^2(\arctan x) = 1 + x^2$.



4. Use the results of problems 2 and 3 above to show that $\frac{d}{dx} \arctan x = \frac{1}{1+x^2}$.