Math 1300: Calculus I **Fall 2020**

Lecture: Section 5.1: Areas and Distances

Lecturer: Sarah Arpin

Today's Goal: Estimating areas

Logistics: We will star this Friday, and finish on Monday. Tuesday will be a *review day* for the upcoming Quiz 6. Also remember, in the distance we have a double quiz day (Nov. 24th has two quizzes) and these are basically replacements of a midterm - they are cumulative and there are no resubmissions.

Warm-Up 1.1 *If $f'(x) = -2\cos(x) + e^{2x}$, which of the following functions is a possibility for $f(x)$?*

(A) $-2\sin(x) + \frac{1}{2}e^{2x} - 3$

(B) $-2\sin(x) + \frac{e^{2x}}{2} + e$

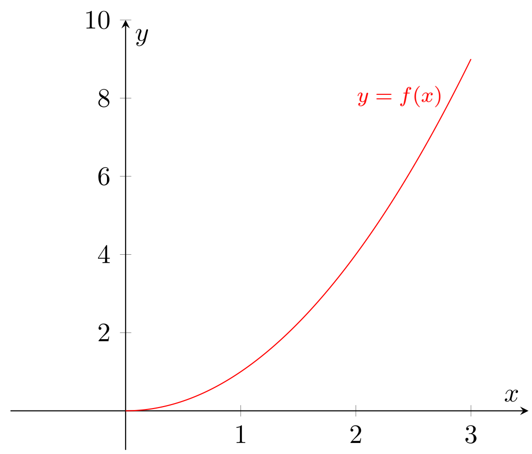
(C) $3 - 2\sin(x) + \frac{1}{2}e^{2x}$

(D) All of the above

(E) None of the above

We will begin developing a technique for finding the area of a shape given by a curve. Later on we will relate this back to the antiderivatives we have been computing.

1.1 Area



1.2 Options for Estimating

1.2.1 Choosing the rectangle width

Usually, we will want our rectangles to be the same widths (not always - depends on what information we are given). The ideal way to choose this width is to say...
If you want n rectangles to cover the x-axis interval $[a, b]$, then each interval should be width $\frac{b-a}{n}$.

For example, if you want to draw 5 rectangles to cover the x-axis interval $[0, 3]$, each rectangle would be...

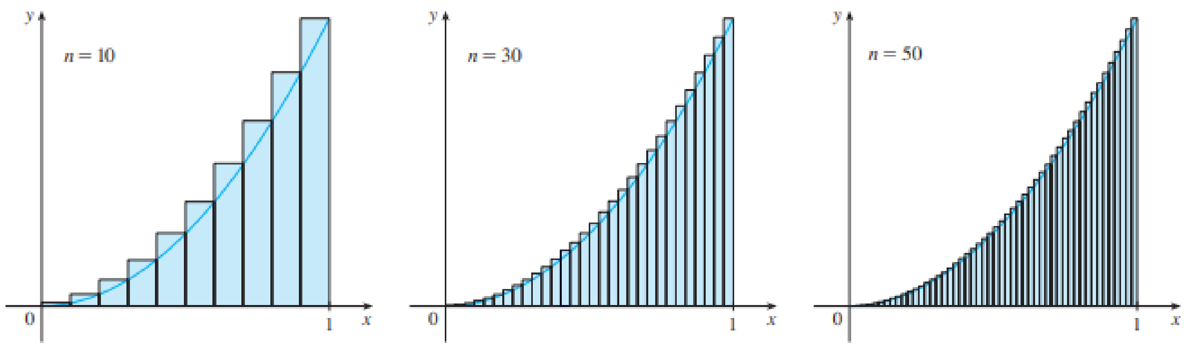


FIGURE 8

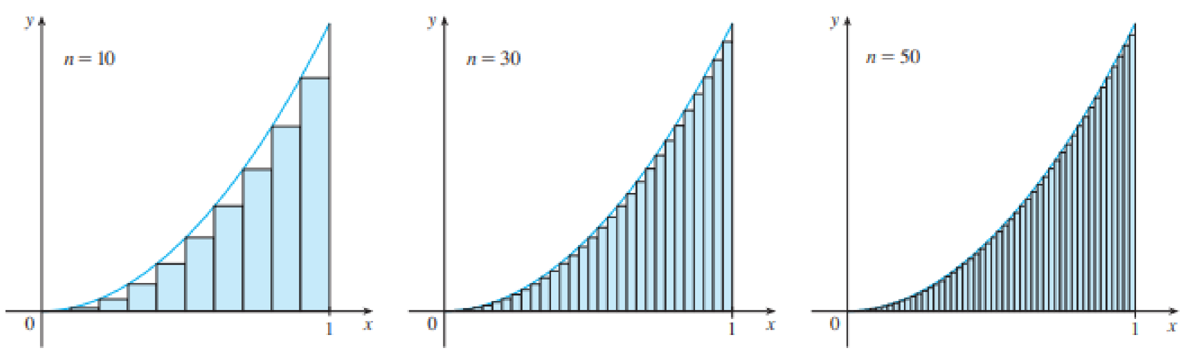


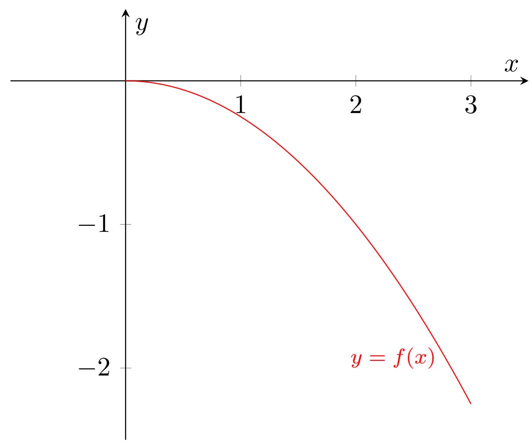
FIGURE 9 The area is the number that is smaller than all upper sums and larger than all lower sums

1.2.2 Choosing the rectangle height

Once you set the intervals along the x-axis (basically picking the number of rectangles you are going to use), now you need to chose how high you draw each rectangle. Here are some common options:

1. Right endpoint of each x-axis interval
2. Left endpoint of each x-axis interval
3. Midpoint of each x-axis interval

1.2.3 What if the curve goes below the x-axis?



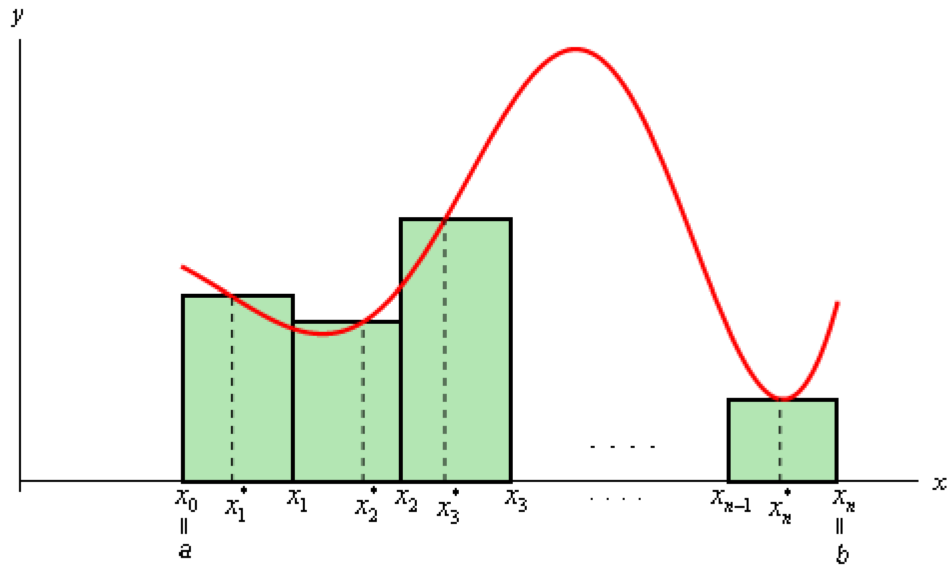
1.2.4 Examples

Example 1.2 *Estimate the area under the curve* $y = \sqrt{x-1}$ *betwee* $x = 1$ *and* $x = 5$ *using four rectangles with the heights calculated from the right endpoints. Repeat the exercise using heights calculated from left endpoints and compare the two. Make two sketches to accompany your calculations.*

Example 1.3 *The speed of a runner is measured at intervals during the first few minutes of a race. Estimate the distance that the runner travels in the first three minutes using these measurements.*

Time (min.)	Speed (meters per min.)
1	150
1.5	175
2.5	165
3	160

1.3 Formal Definition



The area A between the curve $f(x)$ and the x-axis can be estimated using the area of n rectangles, each of width Δx, using the formula:

$$A_n \approx f(x_1^*)\Delta x + f(x_2^*)\Delta x + f(x_3^*)\Delta x + \cdots + f(x_n^*)\Delta x$$

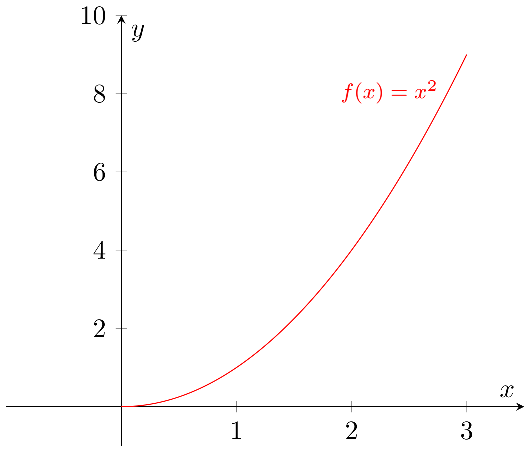
As the width of these rectangles approaches 0, our approximation approaches the true are A:

$$A = \lim_{\Delta x \to 0} A_n = \lim_{\Delta x \to 0} (f(x_1^*)\Delta x + f(x_2^*)\Delta x + \cdots + f(x_n^*)\Delta x)$$

Using a special notation called **summation notation**, this can be expressed concisely:

$$A = \lim_{n \to \infty} \sum_{i=1}^{n} f(x_i^*)\Delta x$$

Example 1.4 *Find the exact area under the curve $f(x) = x^2$ between $x = 1$ and $x = 3$ using a limit of Riemann sums.*



1.4 Cool Facts

$$1 + 2 + \cdots + n = \sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

$$1^2 + 2^2 + \cdots + n^2 = \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

$$1^3 + 2^3 + \cdots + n^3 = \sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$$