4.8

Wednesday, November 4, 2020 12:36 PM

Math 1300: Calculus I **Fall 2020**

Lecture: Section 4.8: Antiderivatives

Lecturer: Sarah Arpin

Today's Goal: Learn about antiderivatives!

Logistics: We will start and finish this section on Wednesday. Don't forget Check-In 14 on Friday! Review sections 4.6 and 4.8 (this section!).

Warm-Up 1.1 *Find two positive numbers whose sum is 300 and whose product is a maximum.*

$x+y=300$

$\ast$ $\boxed{y=300-x}$

$\boxed{x \cdot y = \text{product}}$

$p(x,y) = xy$

$p(x) = x(300-x)$

Domain $[0, 300]$ $\to p(x) = 300x - x^2$

$p'(x) = 300 - 2x = 0$

$2x = 300$

$x = 150$

$p''(x) = -2 \to p$ is con. down

$x = 150$ is location of a max. of $p(x)$.

- *(A)* $\boxed{150, 150}$
- *(B)* $100, 200$
- *(C)* $50, 250$
- *(D)* $300, 300$
- *(E)* *None of the above*

1.1 Antiderivatives

1.1.1 Terminology

Find the derivative of $f(x) = x^3 - 2x + 1$:

$$f'(x) = 3x^2 - 2$$

Let's just name this new function $g(x)$:

$$\boxed{g(x) = 3x^2 - 2}$$

Since the derivative of $f(x)$ is equal to $g(x)$, we say that $f(x)$ is an **antiderivative** of $g(x)$.

Question: Are there other anti-derivatives of $g(x)$?

yes! $x^3 - 2x + C$, for any real # C

A few examples: $f(x) = x^3 - 2x + 4$

Theorem 1.2 *If $F(x)$ is an antiderivative of $f(x)$, then so is $F(x) + C$, for any real number C.*

$f(x) = 3x^2 - 2$

$\dashrightarrow x^3 - 2x + C$

Example 1.3 *Find all functions $f(x)$ that satisfy $f'(x) = \frac{2}{1+x^2} - e^{-x}$.*

$\frac{2}{1+x^2}$: this is $\frac{d}{dx}\left(2\arctan(x)\right)$

Recall: $\frac{d}{dx}\left(\arctan(x)\right) = \frac{1}{1+x^2}$

$-e^{-x}$: this is $\frac{d}{dx}\left(e^{-x}\right)$

Recall: $\frac{d}{dx}e^{x} = e^{x}$

$\frac{d}{dx}e^{-x} = e^{-x}\cdot -1 = -e^{-x}$

$$\boxed{f(x) = 2\arctan(x) + e^{-x} + C \text{ for any real \# } C}$$

Example 1.4 *Find $f(x)$ if $f'(x) = 3x^2 - 4x + 5$ and $f(-1) = 2$.*

narrow down from a family of sol's to a single solution

$3x^2$: is $\frac{d}{dx}(x^3)$

$-4x$: is $\frac{d}{dx}(-2x^2)$ Recall: $\frac{d}{dx}(x^2) = 2x$

5 : is $\frac{d}{dx}(5x)$

Notice: The antiderivative of x^n is $\frac{x^{n+1}}{n+1}$

$f(x) = x^3 - 2x^2 + 5x + C$

use $f(-1) = 2$ to find C:

$2 = (-1)^3 - 2(-1)^2 + 5(-1) + C$

$2 = -1 - 2 - 5 + c$

$10 = c$

$$\boxed{f(x) = x^3 - 2x^2 + 5x + 10}$$

Example 1.5 *Find $g(t)$ if $g''(t) = 2e^t + 3\sin(t)$, $g(0) = 0$, and $g(\pi) = 0$.*

First, find $g'(t) = 2e^t - 3\cos(t) + C$

$\frac{d}{dt}(2e^t) = 2e^t$

$\left(\frac{d}{dt}(e^{2t}) = 2e^{2t}\right)$

$$\boxed{\frac{d}{dt}(\cos(t)) = -\sin(t)}$$

$\frac{d}{dt}\sin(t) = \cos(t)$

$g(t) = 2e^t - 3\sin(t) + Ct + D$

where C and D are real #'s

Use $g(0) = 0$:

$0 = 2e^0 - \cancel{3\sin(0)} + C\cdot\cancel{(0)} + D$

$0 = 2 + D$

$D = -2$

use $g(\pi) = 0$:

$0 = 2e^{\pi} - 3\sin(\pi) + C\cdot\pi - 2$

$0 = 2e^{\pi} + C\pi - 2$

$\frac{2 - 2e^{\pi}}{\pi} = C$

$g(t) = 2e^t - 3\sin(t) + \left(\frac{2-2e^{\pi}}{\pi}\right)t - 2$

Recall now and always that the acceleration due to gravity is $-9.8m/s^2$, or equivalently $-32ft/s^2$.

Example 1.6 *A ball is thrown upward with a speed of 48 ft/s from the edge of a cliff 432 ft above the ground. Find its height above the ground t seconds later. When does it reach its maximum height? When does it hit the ground?*

$a(t) = -32$

c real #

$v(t)$ is an antiderivative of $a(t)$

$v(t) = -32t + C$ Find c using: $v(0)=48$

$48 = -32 \cdot 0 + C$

$48 = C$

$v(t) = -32t + 48$

$h(t)$ is an antideriv of $v(t)$

$h(t) = -16t^2 + 48t + C$ (new real # c)

$h(0) = 432$. use to find C:

$432 = -16 \cdot 0^2 + 48 \cdot 0 + C \longrightarrow C = 432$

$$\boxed{h(t) = -16t^2 + 48t + 432}$$

Example 1.7 *A stone was dropped off a cliff and hit the ground with a speed of 120 ft/s. What is the height of the cliff?*