04.06

Monday, November 2, 2020 12:28 PM

Math 1300: Calculus I **Fall 2020**

Lecture: Section 4.6: Optimization

Lecturer: Sarah Arpin

Today's Goal: Use calculus to optimize real-life.

Logistics: We will start and finish these examples today, but do a related ~~example~~ activity Tuesday.
Friday we have Check-In 14! It will cover sections 4.6 and 4.8.

Warm-Up 1.1 *Find the value of* $\lim_{x\to-\infty} xe^{x-1}$. $\rightsquigarrow -\infty\cdot 0$ indet. form

$$\lim_{x\to-\infty} \frac{x}{1/e^{x-1}} = \lim_{x\to-\infty} \frac{x}{e^{-x+1}} \rightsquigarrow \frac{-\infty}{\infty} \quad \text{l'Hop. applies:}$$

$$= \lim_{x\to-\infty} \frac{1}{-e^{-x+1}} \to 0 \quad \left(\frac{1}{-\infty}\right)$$

$$= 0$$

$\boxed{(A)\ 0}$ ✓

(B) $-\infty$

(C) ∞

(D) 1

(E) None of the above.

1.1 Word Problem Tips

(1) Read the problem multiple times, and highlight key points.

(2) Draw a picture. or many!

(3) Define variables.

(4) Write a relevant equation, using derivatives to optimize.

Recall: If f' changes from neg. to pos., this is the location of a local min. on the graph of f.

If $f' = 0$ and $f'' > 0 \rightarrow$ ∪

$\rightarrow$ See 4.3 1st & 2nd deriv. tests.

1.2 Examples

Example 1.2 *A rectangle has its base along the x-axis and its upper two vertices lie on the graph of the parabola $y = 12 - x^2$.*

$y = 12-(2)^2 =$

b)

$4 \times 8 = \boxed{32}$

(a) *Draw a picture of this.*

(b) *When the base of the rectangle has length 4, the area of the rectangle is.*

(c) *Write a function that gives the area of the rectangle in terms of the length of its base.*

(d) *What is the domain of this function.*

~~(e) *What is the domain of this function?*~~

(f) *What is the maximum possible area of the rectangle?*

c)

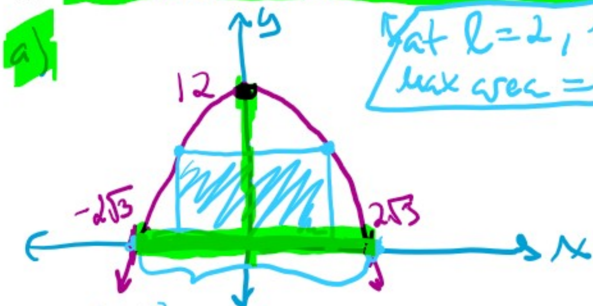
$\ell(12 - \frac{\ell^2}{4})$, " $= 12\ell - \frac{\ell^3}{4}$

$(\frac{\ell}{2})^2 = \frac{\ell^2}{2^2} = \frac{\ell}{2}\cdot\frac{\ell}{2}$

Area $= \ell \cdot (12 - (\frac{\ell}{2})^2)$

$A(\ell) = \ell \cdot (12 - (\frac{\ell}{2})^2)$

a)



$\boxed{\text{at } \ell = 2, \text{ the max area} = 32}$

d) Domain for ℓ:

☆ $[0, 4\sqrt{3}]$

x-int: $0 = 12 - x^2$
$x^2 = 12$
$x = \pm\sqrt{12}$

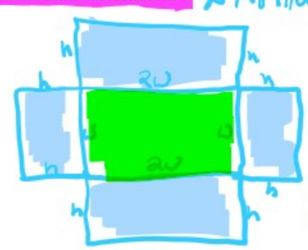
e) Maximize $A(\ell) = 12\ell - \frac{\ell^3}{4}$ on $[0, 4\sqrt{3}]$: $A'(\ell) = 12 - \frac{3}{4}\ell^2 = 0$
$12 = \frac{3}{4}\ell^2$
$16 = \ell^2$
$\pm 4 = \ell$

ℓ	$A(\ell)$
0	0
4	32
$4\sqrt{3}$	0

Example 1.3 *A rectangular storage container with an open top is to have a volume of 10 m. The length of its base is twice the width. Material for the base costs \$10 per square meter. Material for the sides costs \$6 per square meter. Find the cost of materials for the cheapest such container.* ☆ Minimize cost

$V = 10$ m

$2w^2 h = V$
$2w^2 h = 10$
$h = \frac{5}{w^2}$



on $[0, \infty)$, $w = \sqrt[3]{4.5}$ is the location of an abs. minimum:

$C(\sqrt[3]{4.5}) = 20(\sqrt[3]{4.5})^2 + \frac{180}{\sqrt[3]{4.5}}$

Total Area of sides: $2hw + 4hw$

→ Total cost of sides: $6(2hw + 4hw)$

$= 12 \cdot \frac{5}{w^2} \cdot w + 24 \cdot \frac{5}{w^2} \cdot w$

$= \frac{60}{w} + \frac{120}{w} = \frac{180}{w}$

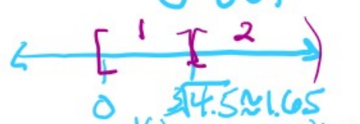
Total cost of base $= \$10 \cdot 2w^2$

Total cost:

$C(w) = 20w^2 + \frac{180}{w}$ ... $180w^{-1}$

Minimize $C(w)$:

$C'(w) = 40w - \frac{180}{w^2} = 0$



$C'(1) < 0$ C dec. $C'(2) > 0$ C inc.

$40w = \frac{180}{w^2}$
$40w^3 = 180$
$w^3 = 4.5$

$w = \sqrt[3]{4.5}$
$w = 0$ is CP too

Example 1.4 *A piece of wire 10 m long is cut into two pieces. One piece is bent into a square and the other is bent into an equilateral triangle. How should the wire be cut so that the total area enclosed is a maximum? A minimum?*

Example 1.5 *During the summer months Terry makes and sells necklaces on the beach. Last summer he sold the necklaces for each and his sales averaged 20 per day. When he increased the price by $1, he found that the average decreased by two sales per day.*

(a) Find the demand function, assuming that it is linear.

(b) If the material for each necklace costs Terry , what should the selling price be to maximize his profit?