04.02 Min/Max Vals

Tuesday, October 20, 2020 12:37 PM

Math 1300: Calculus I **Fall 2020**

Lecture: Section 4.2: Minimum and Maximum Values

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Today's Goal: Minimum and Maximum Values

Logistics: We will start this section with an activity Tuesday and finish Wednesday. There is a check-in Friday, it's set to cover the topics we are covering this week: linear approximation, min/max problems, and extreme value theorem (Sections 3.9 & 4.2).

Warm-Up 1.1 *(Activity is the warm-up - ask students with the ability to easily write on their screens to "raise hand" or something in the participants window to help create groups)*

1.1 Local/Relative vs. Global/Absolute

For the following definitions, let $f(x)$ be a function and let D denote the domain of that function.

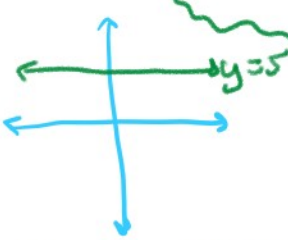
Definition 1.2 (Local or Relative Minima and Maxima) *The value $f(c)$ is a local minimum (resp. maximum) if $f(x) \geq f(c)$ (resp. $f(x) \leq f(c)$) for values of x near c.*

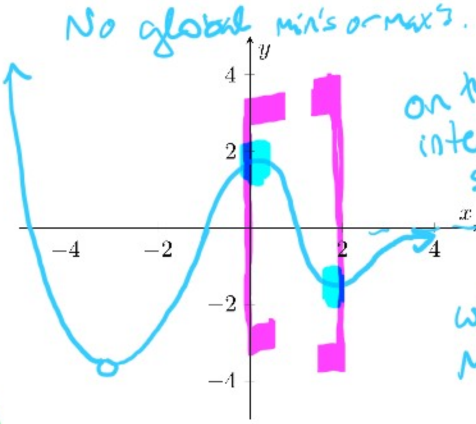
Definition 1.3 (Global or Absolute Minima and Maxima) *The value $f(c)$ is an absolute minimum (resp. maximum) if $f(x) \geq f(c)$ (resp. $f(c) \leq f(c)$) for all values of x in the domain D of $f(x)$.*

Remark 1.4 *These minima/maxima are **y-values**, and we say they **ocurr at** the x-values at which the function attains the value.* "$f(2) = -1$ is a local minimum occurring at $x = 2$"

Remark 1.5 *Not every function has these values. Sketch a function whose domain is $(-\infty, \infty)$ that does not have a global maximum or minimum, but it does have at least one local minimum and maximum.*

No global min's or max's.

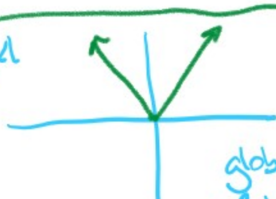




on the closed interval $[0,2]$, since f is cont., the EVT guarantees we will have an abs. min and an abs. max.

Global min = Global max = 5 occurs at every x-val in the domain $(-\infty, \infty)$

$y = |x|$



global max = none
global min = 0, occurs at $x = 0$
no other minima or maxima.

When your domain is a closed interval then you will get abs. min + max on that closed interval domain

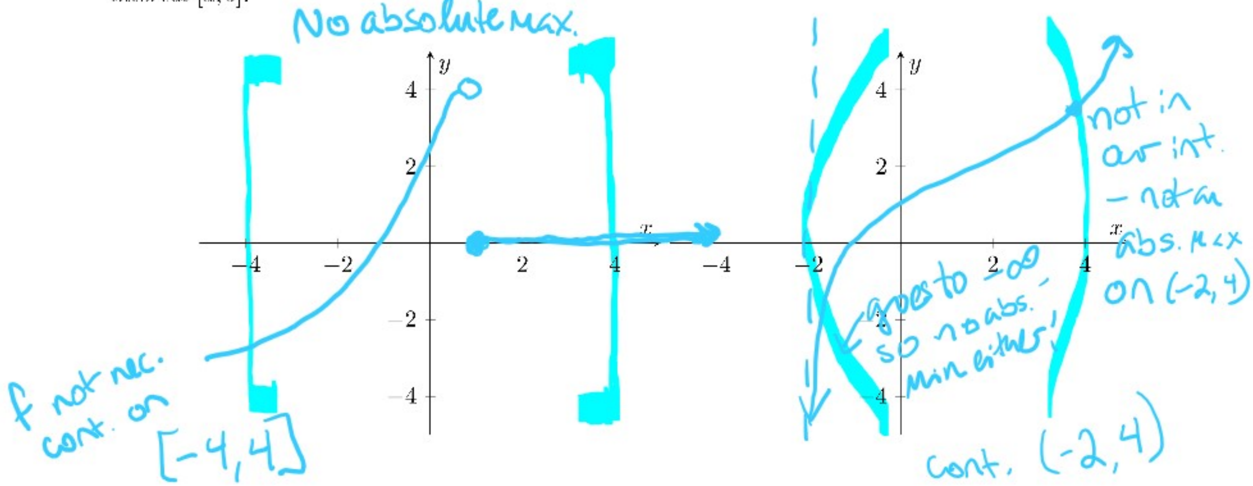
EVT

$f(x)$

Theorem 1.6 (The Extreme Value Theorem) *If is continuous on a closed interval $[a, b]$, then attains an absolute maximum value $f(c)$ and an absolute minimum value $f(d)$ at some x-values c and d in $[a, b]$.*

↑closed

Remark 1.7 *Do we need both hypotheses (continuous, closed interval) of this theorem? Draw a counterexample to the theorem if we do not require f to be continuous, and another one if we do not use a closed interval $[a, b]$.*



1.2 Where do minima/maxima occur?

Theorem 1.8 (Fermat) *If $f(c)$ is a local minimum or maximum and $f'(c)$ exists, then $f'(c) = 0$.*

Definition 1.9 (Critical number) *$x = c$ is a critical value for $f(x)$ if c is in the domain of f and $f'(c) = 0$ or does not exist.*

Example 1.10 *Find the critical number(s) of the function $y = |2x - 1|$.*

@ the vertex $|2x-1|$

Find x-coord. of vertex: $2x-1=0$

$2x=1$

$\boxed{x=\frac{1}{2}}$

Example 1.11 *Find the critical number(s) of the function $g(t) = \frac{t-1}{t^2+4}$.*

$$g'(x) = \frac{(t^2+4)(1)-(t-1)(2t)}{(t^2+4)^2} = \frac{t^2+4-2t^2+2t}{(t^2+4)^2}$$

$$g'(x) = \frac{-t^2+2t+4}{(t^2+4)^2}$$

$\stackrel{?}{=} 0 \longrightarrow -t^2+2t+4=0$

$t = \frac{-2\pm\sqrt{4+4\cdot4}}{-2} = \frac{-2\pm\sqrt{20}}{-2}$

$= \frac{-2\pm2\sqrt{5}}{-2} = \boxed{1\pm\sqrt{5}}$ CN's

? DNE ?

$(t^2+4)^2 \stackrel{?}{=} 0$

$t^2+4 \stackrel{?}{=} 0$

No real value of t makes true

$* \; t^2+4 \geq 4$

1.3 The Closed Interval Method

We will be interested in finding the absolute minima/maxima of a function $f(x)$ on a closed interval $[a, b]$.

Example 1.12 *Find the absolute minima and maxima of $f(x) = x - \ln(x)$ on $[0.5, 2]$.*

CN

(1) Find the values of f at the critical numbers in (a, b): $= (.5, 2)$

$f'(x) = 1 - \frac{1}{x} = \frac{x-1}{x}$

$f'(x) = 0$ when num $= 0$ and denom $\neq 0$

$f'(x)$ DNE when denom $= 0$.

$\boxed{x = 1}$

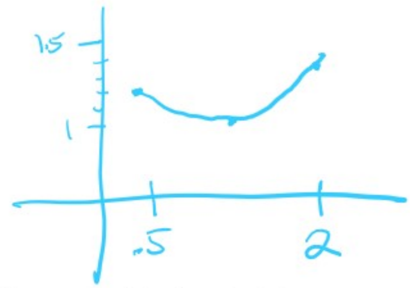
$\boxed{f(1) = 1}$

★ $x = 0$ is a CN, but it's not in $(.5, 2)$

(2) Find the values of f at the endpoints of the interval: $f(a)$, $f(b)$

$f(.5) = .5 - \ln(.5) = 1.19$

$f(2) = 2 - \ln(2) = 1.31$



values of f

(3) The largest value from the above two lists is the absolute maximum. The smallest is the absolute minimum.

Abs. Max is 1.31, occurring at $x = 2$.
Abs. min is 1, occurring at $x = 1$

Example 1.13 *Find the absolute minima and maxima of $f(x) = x - 2\arctan(x)$ on $[0, 4]$.*

(1) Find the values of f at the critical numbers in (a, b): $(0, 4)$

$f'(x) = 1 - 2 \cdot \frac{1}{1+x^2} = \frac{1+x^2}{1+x^2} - \frac{2}{1+x^2}$

$f'(x) = \frac{x^2 - 1}{x^2 + 1}$ $\longrightarrow$ CN's in $(0,4)$: $x = 1$

$f(1) = 1 - 2\arctan(1)$
$= 1 - 2 \cdot \frac{\pi}{4}$
$= 1 - \frac{\pi}{2}$
$= -.57$

(2) Find the values of f at the endpoints of the interval: $f(a)$, $f(b)$

$f(0) = 0 - 2\arctan(0) = 0$

$f(4) = 4 - 2\arctan(4) = 1.35$

(3) The largest value from the above two lists is the absolute maximum. The smallest is the absolute minimum.

Abs. min $= -.57$ occurs at $x = 1$

Abs. max $= 1.35$ occurs at $x = 4$