**Math 1300: Calculus I** **Fall 2020**

# Lecture: Section 4.2: Minimum and Maximum Values

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**Today's Goal: Minimum and Maximum Values**

Logistics: We will start this section with an activity Tuesday and finish Wednesday. There is a check-in Friday, it's set to cover the topics we are covering this week: linear approximation, min/max problems, and extreme value theorem (Sections 3.9 & 4.2).

**Warm-Up 1.1** *(Activity is the warm-up - ask students with the ability to easily write on their screens to "raise hand" or something in the participants window to help create groups)*

## 1.1 Local/Relative vs. Global/Absolute

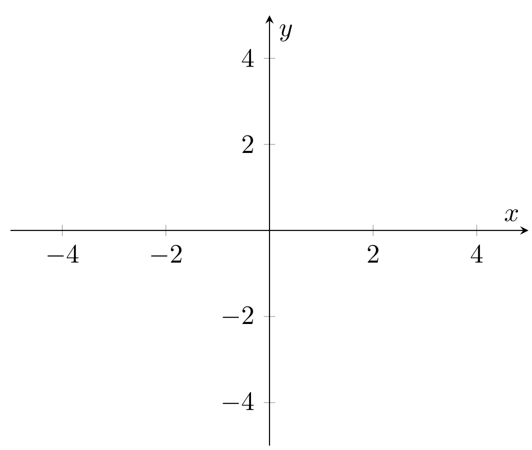
For the following definitions, let $f(x)$ be a function and let $D$ denote the domain of that function.

**Definition 1.2 (Local or Relative Minima and Maxima)** *The value $f(c)$ is a local minimum (resp. maximum) if $f(x) \geq f(c)$ (resp. $f(x) \leq f(c)$) for values of $x$ near $c$.*

**Definition 1.3 (Global or Absolute Minima and Maxima)** *The value $f(c)$ is an absolute minimum (resp. maximum) if $f(x) \geq f(c)$ (resp. $f(c) \leq f(c)$) for all values of $x$ in the domain $D$ of $f(x)$.*

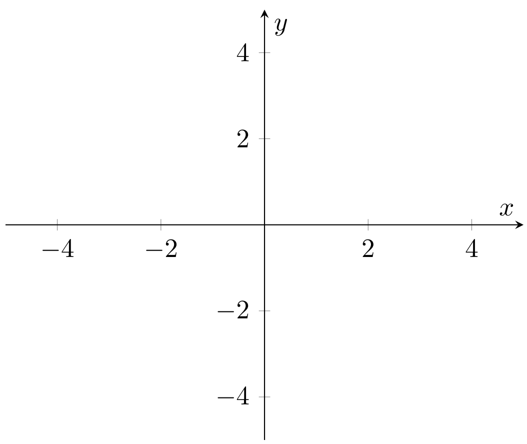
**Remark 1.4** *These minima/maxima are **y-values**, and we say they **ocurr at** the x-values at which the function attains the value.*

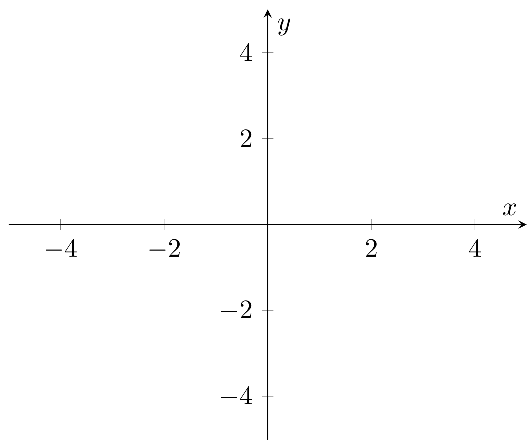
**Remark 1.5** *Not every function has these values. Sketch a function whose domain is $(-\infty, \infty)$ that does not have a global maximum or minimum, but it does have at least one local minimum and maximum.*



**Theorem 1.6 (The Extreme Value Theorem)** *If is continuous on a closed interval $[a, b]$, then attains an absolute maximum value $f(c)$ and an absolute minimum value $f(d)$ at some $x$-values $c$ and $d$ in $(a, b)$.*

**Remark 1.7** *Do we need both hypotheses (continuous, closed interval) of this theorem? Draw a counterexample to the theorem if we do not require $f$ to be continuous, and another one if we do not use a closed interval $[a, b]$.*




## 1.2 Where do minima/maxima occur?

**Theorem 1.8 (Fermat)** *If $f(c)$ is a local minimum or maximum and $f'(c)$ exists, then $f'(c) = 0$.*

**Definition 1.9 (Critical number)** *$x = c$ is a critical value for $f(x)$ if $c$ is in the domain of $f$ and $f'(c) = 0$ or does not exist.*

**Example 1.10** *Find the critical number(s) of the function $y = |2x - 1|$.*

**Example 1.11** *Find the critical number(s) of the function $g(t) = \frac{t-1}{t^2+4}$.*

## 1.3 The Closed Interval Method

We will be interested in finding the absolute minima/maxima of a function $f(x)$ on a closed interval $[a, b]$.

**Example 1.12** *Find the absolute minima and maxima of* $f(x) = x - \ln(x)$ *on* $[0.5, 2]$.

(1) Find the values of $f$ at the critical numbers in $(a, b)$:

(2) Find the values of $f$ at the endpoints of the interval: $f(a)$, $f(b)$

(3) The largest value from the above two lists is the absolute maximum. The smallest is the absolute minimum.

**Example 1.13** *Find the absolute minima and maxima of* $f(x) = x - 2\arctan(x)$ *on* $[0, 4]$.

(1) Find the values of $f$ at the critical numbers in $(a, b)$:

(2) Find the values of $f$ at the endpoints of the interval: $f(a)$, $f(b)$

(3) The largest value from the above two lists is the absolute maximum. The smallest is the absolute minimum.