03.09 Linearization

Monday, October 19, 2020 12:25 PM

*DO HW4.3 Webassign!

Math 1300: Calculus I **Fall 2020**

Lecture: Section 3.9: Linear Approximation

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Today's Goal: Learn how to approximate functions using calculus.

Logistics: We will start and finish this section on Monday. There is a check-in Friday, it's set to cover the topics we are covering this week: linear approximation, min/max problems, and extreme value theorem (Sections 3.9 & 4.2).

Warm-Up 1.1 *Find the equation of the tangent line to the curve* $y = \ln(\sin^2(x) + 1)$ *at* $x = \pi/4$.

point: $y = \ln(\sin^2(\pi/4) + 1)$

$y = \ln\left(\left(\frac{\sqrt{2}}{2}\right)^2 + 1\right)$

$y = \ln\left(\frac{2}{4} + 1\right) = \ln\left(\frac{3}{2}\right)$

$-\ln\left(\frac{2}{3}\right) = \ln\left(\left(\frac{2}{3}\right)^{-1}\right) = \ln\left(\frac{3}{2}\right)$

$\frac{1}{\frac{2}{3}} = \frac{3}{2}$

(A) $y = \frac{3}{2}(x - \pi/4) + \ln(3/2)$

(B) $\boxed{y = \frac{2}{3}(x - \pi/4) + \ln(3/2)}$

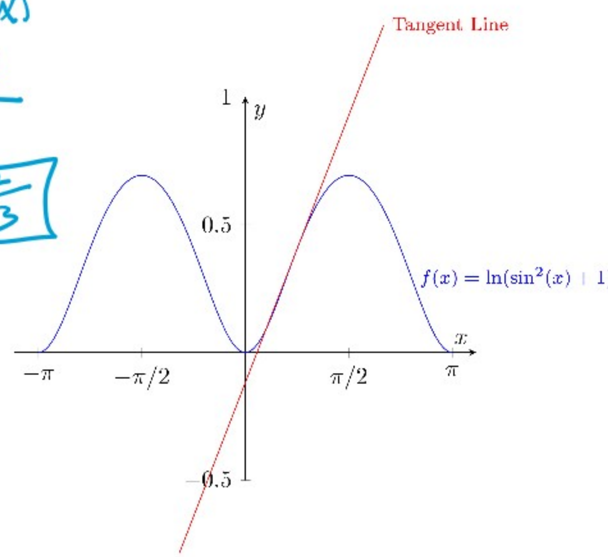
(C) $y = \frac{3}{2}(x - \pi/4) - \ln(2/3)$

(D) $y = \frac{2}{3}(x + \pi/4) + \ln(2/3)$ $y = \ln(\sin^2 x + 1)$

(E) *None of the above.*

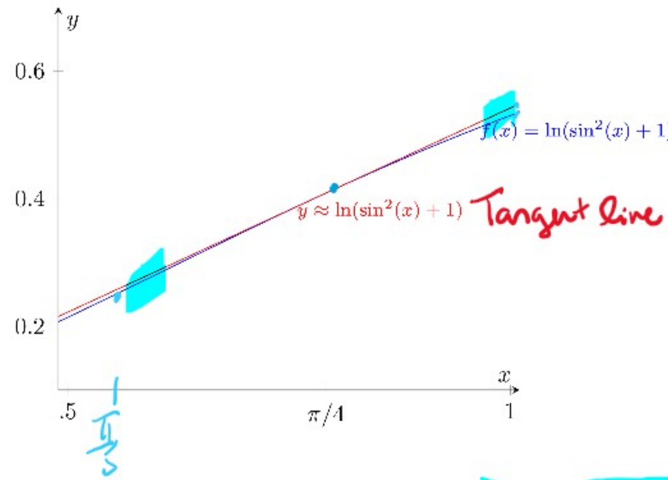
$y' = \frac{1}{\sin^2 x + 1} \cdot 2\sin(x) \cdot \cos(x)$

$y' @ x = \pi/4 = \frac{2 \cdot \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{2}}{2}}{\frac{3}{2}} = \frac{1}{\frac{3}{2}} = \boxed{\frac{2}{3}}$



1.1 Why Tangent Lines?

What can we *use* the tangent line to a curve for?



Close to the point of tangency, the values of the function can be approximated by the values of the tangent line.

Definition 1.2 *To capture this idea of approximation, we call the tangent line to the curve $y = f(x)$ at $x = a$ the* ***linearlization*** *of $f(x)$ at a.*

Example 1.3 *Use the linearlization of $y = \ln(\sin^2(x) - 1)$ at $x = \pi/4$ to estimate the value* $\ln(\sin^2(\pi/5)+1)$.

$y = \frac{2}{3}(x - \frac{\pi}{4}) + \ln(3/2)$

$f(x) = \ln(\sin^2(x) + 1)$

$f(\pi/5)$

$f(\pi/5) \approx$ value of tangent line @ $\pi/5$

$f(\pi/5) \approx \boxed{\frac{2}{3}\left(\frac{\pi}{5} - \frac{\pi}{4}\right) + \ln(3/2)}$

Example 1.4 *Find the linearization of $y = \sqrt[5]{x+1}$ at $a = 1$ and use it to approximate the value* $\sqrt[5]{2.1}$

Tangent line at $(1, \sqrt[5]{2})$

what x-val do I plug into y to get this number? Take that x-val and plug into the linearization

Find slope: $y = (x+1)^{1/5}$

$y' = \frac{1}{5}(x+1)^{-4/5} \cdot 1$

slope @ $x = 1$:

$y' = \frac{1}{5} \cdot (2)^{-4/5}$

$= \frac{1}{5\sqrt[5]{16}}$

linearization:

$\boxed{L(x) = \frac{1}{5\sqrt[5]{16}}(x-1) + \sqrt[5]{2}}$

$L(1.1) \approx y$ at $x = 1.1$

$L(1.1) \approx \sqrt[5]{2.1}$

$L(1.1) = \frac{1}{5\sqrt[5]{16}}(1.1 - 1) + \sqrt[5]{2}$

$= \boxed{\frac{.1}{5\sqrt[5]{16}} + \sqrt[5]{2} \approx \sqrt[5]{2.1}}$

Example 1.5 *Use the linearization of some function to estimate the value of* $\ln(1.1)$ *without using a calculator. Justify your choice of function and the location of the linearization.*

Function to "linearize": $f(x) = \ln(x)$

Where to linearize: $x = 1$

Linearization: Point: $(1, \ln(1)$ = exp. you raise e to, to get 1$)$

$(1, 0)$

Slope: $f'(x) = \frac{1}{x}$

$f'(1) = 1$

$L(x) = 1(x-1) + 0$

$L(x) = x - 1$

To estimate $\ln(1.1) = f(1.1) \approx L(1.1)$

$L(1.1) = 1.1 - 1$

$= \boxed{0.1 \approx \ln(1.1)}$

$\ln(e) = 1 \longleftrightarrow e^1 = e$ "$\ln(x)$" $\rightarrow e^{\sqcup} = x$

$\ln(1) = \longleftrightarrow e^{\sqcup} = 1$