03.09

Monday, October 19, 2020 12:25 PM

Math 1300: Calculus I **Fall 2020**

Lecture: Section 3.9: Linear Approximation

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Today's Goal: Learn how to approximate functions using calculus.

Logistics: We will start and finish this section on Monday. There is a check-in Friday, it's set to cover the topics we are covering this week: linear approximation, min/max problems, and extreme value theorem (Sections 3.9 & 4.2).

Warm-Up 1.1 *Find the equation of the tangent line to the curve* $y = \ln(\sin^2(x) + 1)$ *at* $x = \pi/4$.

point: $y = \ln(\sin^2(\pi/4) + 1)$

$y = \ln((\frac{\sqrt{2}}{2})^2 + 1)$

$y = \ln(\frac{2}{4} + 1) = \ln(\frac{3}{2})$

$-\ln(\frac{2}{3}) = \ln((\frac{2}{3})^{-1}) = \ln(\frac{3}{2})$

$\frac{1}{\frac{2}{3}} = \frac{3}{2}$

(A) $y = \frac{3}{2}(x - \pi/4) + \ln(3/2)$

$\boxed{(B)\ y = \frac{2}{3}(x - \pi/4) + \ln(3/2)}$

(C) $y = \frac{3}{2}(x - \pi/4) - \ln(2/3)$

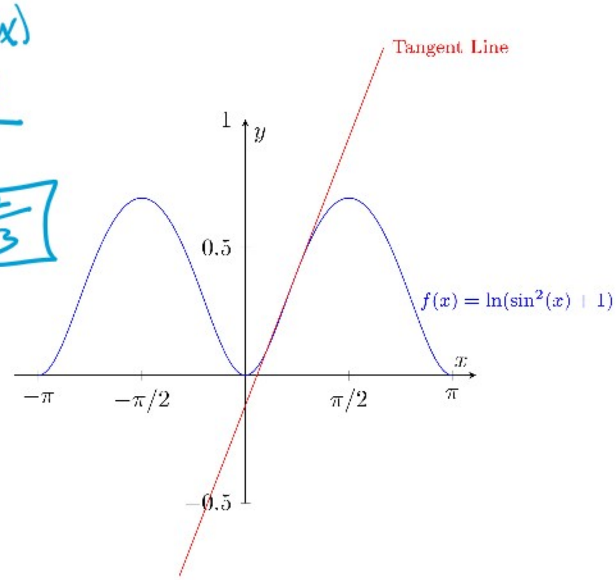
$y = \ln(\sin^2 x + 1)$ (D) $y = \frac{2}{3}(x + \pi/4) + \ln(2/3)$

(E) *None of the above.*

$y' = \frac{1}{\sin^2 x + 1} \cdot 2\sin(x) \cdot \cos(x)$

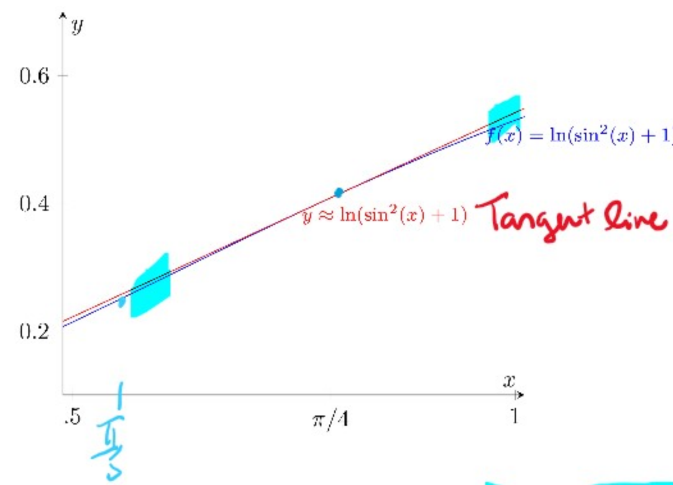
$y' @ x = \pi/4 = \frac{\cancel{2} \cdot \frac{\sqrt{2}}{\cancel{2}} \cdot \frac{\sqrt{2}}{2}}{\frac{3}{2}}$

$= \frac{1}{3/2} = \boxed{\frac{2}{3}}$



1.1 Why Tangent Lines?

What can we *use* the tangent line to a curve for?



Close to the point of tangency, the values of the function can be approximated by the values of the tangent line.

Definition 1.2 *To capture this idea of approximation, we call the tangent line to the curve $y = f(x)$ at $x = a$ the* ***linearlization*** *of $f(x)$ at a.*

Example 1.3 *Use the linearlization of $y = \ln(\sin^2(x) - 1)$ at $x = \pi/4$ to estimate the value $\ln(\sin^2(\pi/5)+1)$.*

$$y = \frac{2}{3}\left(x - \frac{\pi}{4}\right) + \ln(3/2)$$

$$f(x) = \ln(\sin^2(x) + 1)$$

$$f(\pi/5)$$

$f(\pi/5) \approx$ value of tangent line @ $\pi/5$

$$f(\pi/5) \approx \boxed{\frac{2}{3}\left(\frac{\pi}{5} - \frac{\pi}{4}\right) + \ln(3/2)}$$

Example 1.4 *Find the linearization of $y = \sqrt[5]{x+1}$ at $a = 1$ and use it to approximate the value $\sqrt[5]{2.1}$*

Example 1.5 *Use the linearization of some function to estimate the value of* $\ln(1.1)$ *without using a calculator. Justify your choice of function and the location of the linearization.*