03.06 Inverse Trig Derivatives

Monday, October 5, 2020 12:31 AM

Math 1300: Calculus I **Fall 2020**

Lecture: Section 3.6: Inverse Trig Functions

Lecturer: Sarah Arpin

Today's Goal: We start this on Wednesday, finish on Friday.

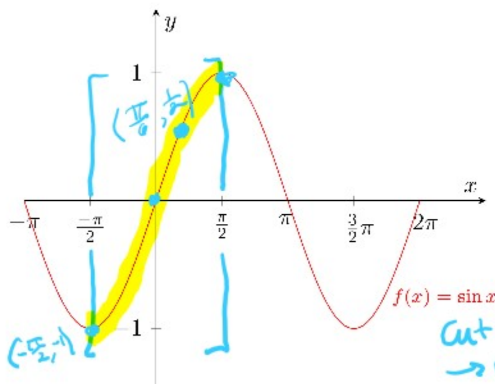
Logistics: Friday there is a check-in!

Warm-Up 1.1 *True or False:* $\frac{d}{dx}\ln(10) = \frac{1}{10}$

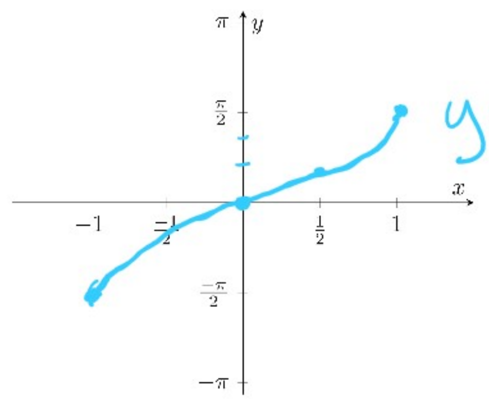
1.1 $Arcsin(x)$ $= \sin^{-1}(x) \neq \frac{1}{\sin(x)}$

inverse as a function

Domain → Range (sin(x)); Angles ← "O/H" (arcsin(x))



Cut off domain of sin(x) → restricting range of arcsin(x)



y = arcsin(x) — obtained from flipping (x, y) on y = sin x to (y, x)

$f(f^{-1}(y)) = y$

$f^{-1}(f(x)) = x$

Recall how to do computations with inverse trig functions:

$$\cos(\arcsin(\frac{-1}{2})) =$$

$\sin(\theta) = \frac{-1}{2}$ and θ needs to be in $[-\pi/2, \pi/2]$

$\cos(-\frac{\pi}{6}) = \frac{\sqrt{3}}{2}$

To find the derivative of $\arcsin(x)$, we will use a property of inverse function along with implicit differentiation, and some properties of trig functions. The necessary property:

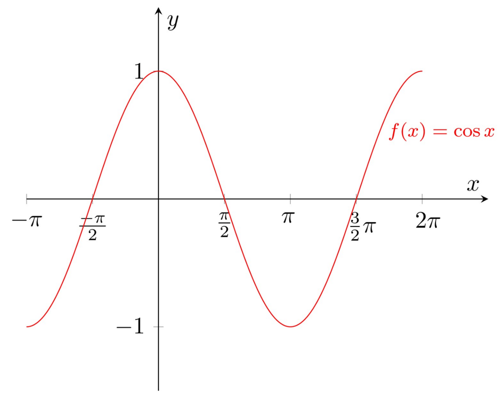
$$\sin(\arcsin(x)) = x$$

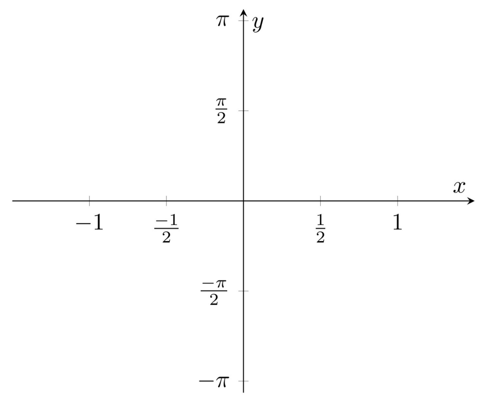
Note that this only holds for x in the interval:

In summary:

$$\frac{d}{dx}\arcsin(x) = \frac{1}{\sqrt{1-x^2}}$$

1.2 *Arccos*(x)





Again with the domain restriction in mind, we can use a similar process to find the derivative of $\arccos(x)$:

$$\cos(\arccos(x)) = x$$

1.3 *Arctan(x)*

$$\frac{d}{dx}\arctan(x) = \frac{1}{1+x^2}, \text{ on the domain of } \arctan(x) \text{ which is } -\pi/2 < x < \pi/2$$

1.4 Examples

Evaluate:

Example 1.2 $\csc(\arccos(3/5)) =$

Example 1.3 $\cos(\arcsin(1/2)) =$

Find the derivatives of the following functions:

Example 1.4 $y = \arctan\left(\frac{x^2-1}{x^2+2}\right)$

Example 1.5 $f(x) = e^{\arcsin(x^2)+3x+1}$