# 9-28

Monday, September 28, 2020 12:24 PM

**Math 1300: Calculus I** **Fall 2020**

## Lecture: Section 3.3: Derivatives of Trig Functions

*Lecturer: Sarah Arpin*

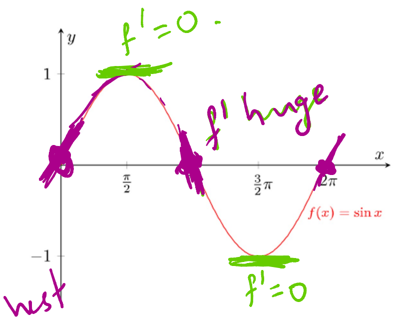
**Today's Goal: Learn derivatives of trig functions.**

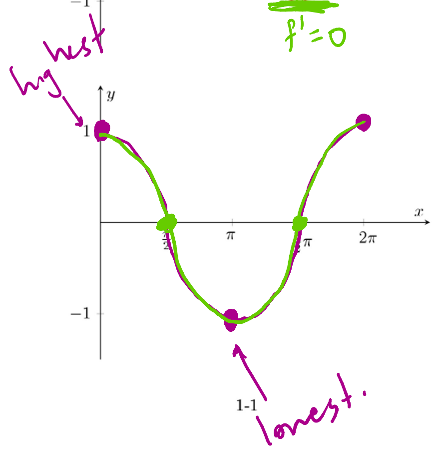
Logistics: We should be starting and finishing this section on a Monday.
There is an evening quiz tomorrow! It covers section 2.8 and 3.1 - 3.3.

**Warm-Up 1.1** *Find $f'(1)$ for $f(x) = (2x-1)(e^x + x)$.*

### 1.1 Graphically

Let's look at the $f(x) = \sin(x)$ function and see if we can make some remarks about what we expect the graph of the derivative to look like:




cos x.

1-1



### 1.2 Some Necessary Trig

Recall the values of the trig function:

| $x$ | 0 | $\frac{\pi}{6}$ | $\frac{\pi}{4}$ | $\frac{\pi}{3}$ | $\frac{\pi}{2}$ |
|---|---|---|---|---|---|
| $\sin(x)$ | 0 | $\frac{1}{2}$ | $\frac{\sqrt{2}}{2}$ | $\frac{\sqrt{3}}{2}$ | 1 |
| $\cos(x)$ | 1 | $\frac{\sqrt{3}}{2}$ | $\frac{\sqrt{2}}{2}$ | $\frac{1}{2}$ | 0 |
| $\tan(x)$ | 0 | $1/\sqrt{3}$ | 1 | $\sqrt{3}$ | $\infty$? |
| $\csc(x)$ | $\infty$ | 2 | $\sqrt{2}$ | $2/\sqrt{3}$ | 1. |

$\tan = \frac{\sin}{\cos}$.

$\frac{0}{2}$ $\frac{\sqrt{1}}{2}$ $\frac{\sqrt{2}}{2}$ $\frac{\sqrt{3}}{2}$ $\frac{\sqrt{4}}{2}$.

| | | | | | |
|---|---|---|---|---|---|
| tan($x$) | 0 | $1/\sqrt{3}$ | 1 | $\sqrt{3}$ | $\infty$? |
| csc($x$) | $\infty$ | 2 | $\sqrt{2}$ | $2/\sqrt{3}$ | 1 |
| sec($x$) | 1 | $2/\sqrt{3}$ | $\sqrt{2}$ | 2 | $\infty$ |
| cot($x$) | $\infty$ | $\sqrt{3}$ | 1 | $1/\sqrt{3}$ | 0 |

And some identities:

$$\sin(a+b) = \sin(a)\cos(b) + \sin(b)\cos(a)$$

$$\lim_{x\to 0} \frac{\sin(x)}{x} = 1$$

$$\lim_{x\to 0} \frac{\cos(x)-1}{x} = 0$$

With these tools, we will be able to algebraically determine the derivative of $\sin(x)$:

$$\frac{d}{dx}\sin(x) = \lim_{h\to 0} \frac{\sin(x+h) - \sin(x)}{h}$$

$$= \lim_{h\to 0} \frac{\sin(x)\cos(h) + \sin(h)\cos(x) - \sin(x)}{h}$$

In conclusion:

$$\frac{d}{dx}\sin(x) =$$



Similarly, we can prove the derivatives of the rest of the trig functions:

| | |
|---|---|
| $\frac{d}{dx}\sin(x) =$ | $\cos(x)$ |
| $\frac{d}{dx}\cos(x) =$ | $-\sin(x)$ |
| $\frac{d}{dx}\tan(x) =$ | $\sec^2(x)$ |
| $\frac{d}{dx}\csc(x) =$ | $-\csc(x)\cot(x)$ |
| $\frac{d}{dx}\sec(x) =$ | $\sec(x)\tan(x)$ |
| $\frac{d}{dx}\cot(x) =$ | $-\csc^2(x)$ |

**Example 1.2** *(1) If $f(x) = \sec(x)$, find $f'(x)$ and $f''(x)$.*

*(2) At what value(s) of $x$ does $f(x) = e^x\cos(x)$ have a horizontal tangent line?*

*(3) Find the equation of the tangent line to the curve $y = \tan(x)$ at $x = \pi/4$.*

*(3) Find the equation of the tangent line to the curve* $y = \tan(x)$ *at* $x = \pi/4$.

*(4) Consider* $f(x) = 2\cos(x) + x$ *on the interval* $0 \leq x \leq 2\pi$. *On what interval(s) is* $f(x)$ *increasing?*