Math 1300: Calculus I **Fall 2020**

Lecture: Section 3.3: Derivatives of Trig Functions

Lecturer: Sarah Arpin

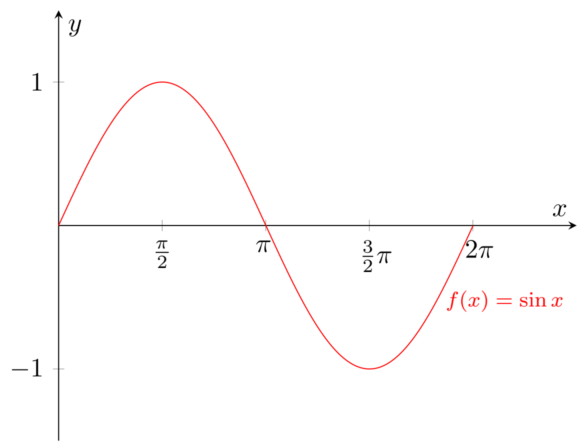
Today's Goal: Learn derivatives of trig functions.

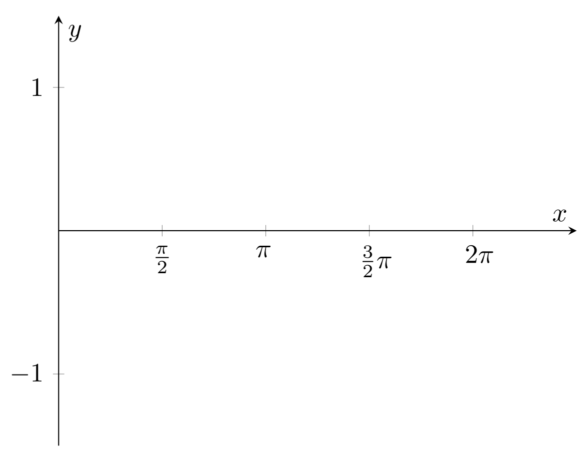
Logistics: We should be starting and finishing this section on a Monday.
There is an evening quiz tomorrow! It covers section 2.8 and 3.1 - 3.3.

Warm-Up 1.1 *Find $f'(1)$ for $f(x) = (2x-1)(e^x + x)$.*

1.1 Graphically

Let's look at the $f(x) = \sin(x)$ function and see if we can make some remarks about what we expect the graph of the derivative to look like:





1.2 Some Necessary Trig

Recall the values of the trig function:

x	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\sin(x)$					
$\cos(x)$					
$\tan(x)$					
$\csc(x)$					
$\sec(x)$					
$\cot(x)$					

And some identities:

$$\sin(a+b) = \sin(a)\cos(b) + \sin(b)\cos(a)$$
$$\lim_{x\to 0}\frac{\sin(x)}{x} = 1$$
$$\lim_{x\to 0}\frac{\cos(x)-1}{x} = 0$$

With these tools, we will be able to algebraically determine the derivative of $\sin(x)$:

$$\begin{aligned}\frac{d}{dx}\sin(x) &= \lim_{h\to 0}\frac{\sin(x+h)-\sin(x)}{h}\\ &= \lim_{h\to 0}\frac{\sin(x)\cos(h)+\sin(h)\cos(x)-\sin(x)}{h}\end{aligned}$$

In conclusion:

$$\frac{d}{dx}\sin(x) =$$

Similarly, we can prove the derivatives of the rest of the trig functions:

$\frac{d}{dx}\sin(x) =$	$\cos(x)$
$\frac{d}{dx}\cos(x) =$	$-\sin(x)$
$\frac{d}{dx}\tan(x) =$	$\sec^2(x)$
$\frac{d}{dx}\csc(x) =$	$-\csc(x)\cot(x)$
$\frac{d}{dx}\sec(x) =$	$\sec(x)\tan(x)$
$\frac{d}{dx}\cot(x) =$	$-\csc^2(x)$

Example 1.2 *(1) If $f(x) = \sec(x)$, find $f'(x)$ and $f''(x)$.*

(2) At what value(s) of x does $f(x) = e^x\cos(x)$ have a horizontal tangent line?

(3) Find the equation of the tangent line to the curve $y = \tan(x)$ at $x = \pi/4$.

(4) Consider $f(x) = 2\cos(x) + x$ on the interval $0 \le x \le 2\pi$. On what interval(s) is $f(x)$ increasing?