03.01 Derivatives of Polynomials and Exponential Functions

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Math 1300: Calculus I **Fall 2020**

Lecture: Section 3.1: Derivatives of polynomials and exponential functions

Lecturer: Sarah Arpin

Today's Goal: Avoid using the limit definition of derivative in some cases

Logistics: We should be starting this on a Monday and finishing on a Tuesday. ~ Tuesday there's an activity
On the Monday we have a **check-in**!

f increasing

Warm-Up 1.1 *If* $\boxed{f'(x) > 0}$ *and* $\boxed{f''(x) < 0}$, *then* f *is:*

concave down

(A) *increasing and concave up.*

(B) *increasing and concave down.*

(C) *decreasing and concave up.*

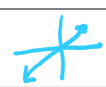
(D) *decreasing and concave down.*

(E) *None of the above.*

$$f'(x) = \lim_{h\to 0} \underbrace{\frac{f(x+h)-f(x)}{h}}_{\text{slope}}$$

$$f'(a) = \lim_{h\to 0}\frac{f(a+h)-f(a)}{h} = \lim_{b\to a}\frac{f(b)-f(a)}{b-a}$$

1.1 Derivative Rules

Rule	Formula	Example
Constant Rule → for any function just a constant #	$f(x)=c$ $f'(x)=0$	$\frac{d}{dx}(5)=0$ $\quad \frac{d}{dx}(\pi^e-42)=0$
x 	$f(x)=x$ $f'(x)=1$	
Power Rule	$f(x)=x^n$ $f'(x)=nx^{n-1}$	$\frac{d}{dx}(x^{24})=24x^{23}$, $\frac{d}{dx}(x^{\pi})=\pi x^{\pi-1}$
Constant Multiple Rule for any real #c: (follows from: $\lim_{x\to a} cf(x) = c\cdot\lim_{x\to a} f(x)$)	$\frac{d}{dx}(c\cdot f(x))$ $=c\frac{d}{dx}f(x)$	$\frac{d}{dx}(3x)=3$ | $\frac{d}{dx}(2x^{-3})=2\frac{d}{dx}(x^{-3})$ $=2(-3x^{-4})$ $=-6x^{-4}$
Sum Rule $\lim_{x\to a}(f(x)+g(x))=\lim_{x\to a}f(x)+\lim_{x\to a}(g(x))$	$\frac{d}{dx}(f(x)+g(x))$ $=f'(x)+g'(x)$	$\frac{d}{dx}(3x^2+x^{1/2})=6x+\frac{1}{2}x^{-1/2}$
Difference Rule	$\frac{d}{dx}(f(x)-g(x))$ $=f'(x)-g'(x)$	$\frac{d}{dx}(\frac{1}{3}x^6-4)=2x^5$
NO PRODUCT OR QUOTIENT RULES		

$$\frac{d}{dx}(f(x)\cdot g(x)) \neq f'(x)\cdot g'(x)$$

NOOOOOO

$\binom{n}{k} = \frac{n!}{(n-k)!\,k!}$

1.1.1 Proof of Power Rule

Gives us a derivative of $f(x) = x^n$ where n is any real number.

$f(x) = x^n$

$f'(x) = \lim_{h\to 0} \frac{(x+h)^n - \cancel{x^n}}{h}$ need to expand

$(x+h)^n = \sum_{i=0}^{n} \binom{n}{i} x^i h^{n-i}$ Binomial expansion

$= \cancel{x^n} + \binom{n}{1}x^{n-1}h + \binom{n}{2}x^{n-2}h^2 + \dots$ higher powers of h

$= \lim_{h\to 0} \frac{n x^{n-1} h + \binom{n}{2} x^{n-2} h^2 + \dots}{h}$

$= \lim_{h\to 0} n x^{n-1} + \underbrace{\binom{n}{2} x^{n-2} h + \dots \text{other } h \text{ terms}}_{\to 0 \text{ b/c } h \to 0}$

$\boxed{\frac{d}{dx}(x^n) = n x^{n-1}}$ ☆

1.1.2 Proof of Sum Rule

Recall: $(f+g)'(x) = f'(x) + g'(x)$

$(f+g)'(x) = \lim_{h\to 0} \frac{(f+g)(x+h) - (f+g)(x)}{h}$

$= \lim_{h\to 0} \frac{f(x+h) + g(x+h) - (f(x) + g(x))}{h}$

$= \lim_{h\to 0} \frac{[f(x+h) - f(x)] + [g(x+h) - g(x)]}{h}$ ☆ $\frac{A+B}{C} = \frac{A}{C} + \frac{B}{C}$

$= \lim_{h\to 0} \left(\frac{f(x+h) - f(x)}{h} + \frac{g(x+h) - g(x)}{h} \right)$

$= \underbrace{\lim_{h\to 0} \left(\frac{f(x+h) - f(x)}{h} \right)}_{} + \underbrace{\lim_{h\to 0} \left(\frac{g(x+h) - g(x)}{h} \right)}_{}$

$= f'(x) + g'(x)$ ✓

What goes wrong with $f \cdot g$? $\neq f'(x)g'(x)$

$(f \cdot g)'(x) = \lim_{h\to 0} \frac{(f\cdot g)(x+h) - (f \cdot g)(x)}{h}$

mult.

$= \lim_{h\to 0} \frac{f(x+h)g(x+h) - f(x)g(x)}{h}$.

Stuck! I'll show you how to get un-stuck in 3.2

*you can go to the MARC for quiz resubs now!

1.2 Examples

Find the derivatives of the following functions using the rules:

(1) $f(x) = 3x^4 - 6x^{1/3} + 2x - 1$

$\boxed{f'(x) = 12x^3 - 2x^{-2/3} + 2} \rightsquigarrow \boxed{f'(x) = 12x^3 - \frac{2}{\sqrt[3]{x^2}} + 2}$

* (2) $f(x) = \underbrace{\sqrt{x}}_{x^{1/2}} + 34x - \underbrace{\frac{1}{x}}_{x^{-1}} = x^{1/3} + 34x - x^{-1}$

$\rightarrow f'(x) = \frac{1}{2}x^{-1/2} + 34 - (-1x^{-2})$

$\boxed{f'(x) = \frac{1}{2\sqrt{x}} + 34 + \frac{1}{x^2}}$

$\frac{A+B}{C} = \frac{A}{C} + \frac{B}{C}$

(3) $f(x) = \frac{x^3 - 4x^2 + \sqrt{x}}{x}$

$\frac{x^3}{x} - \frac{4x^2}{x} + \frac{\sqrt{x}}{x} \rightsquigarrow \frac{x^{1/2}}{x^1}$

$f(x) = x^2 - 4x + x^{-1/2}$ *

$\boxed{f'(x) = 2x - 4 - \frac{1}{2}x^{-3/2}}$

(4) $f(x) = \frac{2x^3 - \sqrt[3]{x^2}}{2}$

$f(x) = x^3 - \frac{1}{2}x^{2/3} \rightsquigarrow \boxed{f'(x) = 3x^2 - \frac{1}{3}x^{-1/3}}$

$\frac{1}{2} \cdot \frac{2}{3} = \frac{2}{6} = \frac{1}{3}$

$(2x)^{1/2}$

(5) $f(x) = \sqrt{2x} + \sqrt{5x}$

$f(x) = 2^{1/2} \cdot x^{1/2} + 5^{1/2} \cdot x^1$

$f'(x) = 2^{1/2}\left(\frac{1}{2}x^{-1/2}\right) + 5^{1/2} \cdot (1) \rightsquigarrow \boxed{f'(x) = \frac{\sqrt{2}}{2\sqrt{x}} + \sqrt{5}}$

(6) $f(x) = 3x^2 - \pi^2$

$\boxed{f'(x) = 6x}$

Common mistake:

$\cancel{f'(x) = 6x - 2\pi}$

Don't make this mistake! π is a constant.

1.3 Exponential Functions

$: e^x, 2^x, \left(\frac{1}{2}\right)^x, \ldots$

You may recall e being vaguely defined in Precalculus: $e \approx 2.718\ldots$. The definition of e involves **limits**, so we are ready for it now!

help us find $\frac{d}{dx}e^x$:

Definition 1.2

$$\boxed{e \text{ is the number such that } \lim_{h\to 0}\frac{e^h - 1}{h} = 1}$$

$$\frac{d}{dx}e^x = \lim_{h\to 0}\frac{e^{x+h} - e^x}{h} = \lim_{h\to 0}\frac{e^x e^h - e^x}{h} = \lim_{h\to 0}\frac{e^x(e^h - 1)}{h}$$

$$= e^x$$

1

Definition 1.3 (Definition of Derivative of $y = x^x$)

$$\boxed{\frac{d}{dx}e^x = e^x}$$

Example 1.4 *Compute the derivative of the function $f(x) = 2e^x - 3x^2 + 54e$*

$$\boxed{f'(x) = 2e^x - 6x}$$

Example 1.5 *At what point on the curve $y = 1 + 2e^x - 3x$ is the tangent line parallel to the line $3x - y = 5$?*

Step 1: find $f'(x)$: $f'(x) = 2e^x - 3$

Step 2: "parallel to $3x - y = 5$" means what slope?

$3x - y = 5 \to y = 3x - 5 \to$ slope $= 3$

Step 3: When is tangent line to $f(x)$ slope $= 3$?

$f'(x) = 3$

$2e^x - 3 = 3 \to 2e^x = 6$

$e^x = 3 \to \boxed{\ln(3) = x}$

Step 4: find y:

$y = 1 + 2e^{(\ln(3))} - 3\ln(3)$

$y = 1 + 6 - 3\ln(3)$

$y = 7 - 3\ln(3)$

point:

$\boxed{(\ln(3), 7 - 3\ln(3))}$

$$\frac{d}{dx}2^x = \ln(2)\cdot 2^x$$

in general: $\boxed{\frac{d}{dx}(b^x) = \ln(b)\cdot b^x}$