03.01 Derivatives of Polynomials and Exponential Functions

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Math 1300: Calculus I **Fall 2020**

Lecture: Section 3.1: Derivatives of polynomials and exponential functions

Lecturer: Sarah Arpin

Today's Goal: Avoid using the limit definition of derivative in some cases

Logistics: We should be starting this on a Monday and finishing on a Tuesday. ~ Tuesday there's an activity
On the Monday we have a **check-in**!

f increasing

Warm-Up 1.1 *If* $\boxed{f'(x) > 0}$ *and* $\boxed{f''(x) < 0}$, *then* f *is:*

concave down

(A) *increasing and concave up.*

(B) *increasing and concave down.*

(C) *decreasing and concave up.*

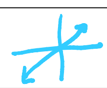
(D) *decreasing and concave down.*

(E) *None of the above.*

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$ slope

$$f'(a) = \lim_{h\to 0}\frac{f(a+h)-f(a)}{h} = \lim_{b\to a}\frac{f(b)-f(a)}{b-a}$$

1.1 Derivative Rules

Rule	Formula	Example
Constant Rule → for any function just = a constant #	$f(x)=c$ $f'(x)=0$	$\frac{d}{dx}(5)=0$ $\quad \frac{d}{dx}(\pi^e-42)=0$
x 	$f(x)=x$ $f'(x)=1$	
Power Rule	$f(x)=x^n$ $f'(x)=nx^{n-1}$	$\boxed{\frac{d}{dx}(x^{24})=24x^{23}}$, $\boxed{\frac{d}{dx}(x^{\pi})=\pi x^{\pi-1}}$
Constant Multiple Rule for any real #c: (follows from: $\lim_{x\to a} c f(x) = c\cdot\lim_{x\to a} f(x)$)	$\frac{d}{dx}(c\cdot f(x))$ $= c\frac{d}{dx}f(x)$	$\frac{d}{dx}(3x)=3$ $\quad \frac{d}{dx}(2x^{-3}) = 2\frac{d}{dx}(x^{-3})$ $= 2(-3x^{-4})$ $= -6x^{-4}$
Sum Rule $\lim_{x\to a}(f(x)+g(x)) = \lim_{x\to a} f(x) + \lim_{x\to a}(g(x))$	$\frac{d}{dx}(f(x)+g(x))$ $= f'(x)+g'(x)$	$\frac{d}{dx}(3x^2+x^{1/2}) = 6x+\frac{1}{2}x^{-1/2}$
Difference Rule	$\frac{d}{dx}(f(x)-g(x))$ $= f'(x)-g'(x)$	$\frac{d}{dx}(\frac{1}{3}x^6-4) = 2x^5$
NO PRODUCT OR QUOTIENT RULES $\frac{d}{dx}(f(x)\cdot g(x)) \neq f'(x)\cdot g'(x)$		

NOOOOOO

$\binom{n}{k} = \frac{n!}{(n-k)!\,k!}$

1.1.1 Proof of Power Rule

Gives us a derivative of $f(x) = x^n$ where n is any real number.

$f(x) = x^n$

$$f'(x) = \lim_{h\to 0} \frac{(x+h)^n - \cancel{x^n}}{h}$$

need to expand

$(x+h)^n = \sum_{i=0}^{n} \binom{n}{i} x^i h^{n-i}$ higher powers of h

"Binomial expansion"

$= \cancel{x^n} + \binom{n}{1} x^{n-1} h + \binom{n}{2} x^{n-2} h^2 + \cdots$

$$= \lim_{h\to 0} \frac{n x^{n-1} h + \binom{n}{2} x^{n-2} h^2 + \cdots}{h}$$

$$= \lim_{h\to 0} n x^{n-1} + \underbrace{\binom{n}{2} x^{n-2} h + \cdots \text{ other } h \text{ terms}}_{\to 0 \text{ b/c } h \to 0}$$

$$\boxed{\frac{d}{dx}(x^n) = n x^{n-1}}$$ ☆

1.1.2 Proof of Sum Rule

Recall: $(f+g)'(x) = f'(x) + g'(x)$

$$(f+g)'(x) = \lim_{h\to 0} \frac{(f+g)(x+h) - (f+g)(x)}{h}$$

$$= \lim_{h\to 0} \frac{f(x+h) + g(x+h) - (f(x) + g(x))}{h}$$

$$= \lim_{h\to 0} \frac{[f(x+h) - f(x)] + [g(x+h) - g(x)]}{h}$$

☆ $\frac{A+B}{C} = \frac{A}{C} + \frac{B}{C}$

$$= \lim_{h\to 0} \left(\frac{f(x+h) - f(x)}{h} + \frac{g(x+h) - g(x)}{h} \right)$$

$$= \underbrace{\lim_{h\to 0} \left(\frac{f(x+h) - f(x)}{h} \right)}_{} + \underbrace{\lim_{h\to 0} \left(\frac{g(x+h) - g(x)}{h} \right)}_{}$$

$$= f'(x) + g'(x) \checkmark$$

What goes wrong with $f \cdot g$? $\neq f'(x) g'(x)$

$$(f \cdot g)'(x) = \lim_{h\to 0} \frac{(f\cdot g)(x+h) - (f \cdot g)(x)}{h}$$

mult.

$$= \lim_{h\to 0} \frac{f(x+h) g(x+h) - f(x) g(x)}{h}$$

Stuck! I'll show you how to get un-stuck in 3.2

1.2 Examples

Find the derivatives of the following functions using the rules:

(1) $f(x) = 3x^4 - 6x^{1/3} + 2x - 1$

(2) $f(x) = \underbrace{\sqrt{x}}_{x^{1/2}} + 34x - \underbrace{\frac{1}{x}}_{x^{-1}} = x^{1/3} + 34x - x^{-1}$

$\rightarrow f'(x) = \frac{1}{2}x^{-1/2} + 34 - (-1x^{-2})$

$\boxed{f'(x) = \frac{1}{2\sqrt{x}} + 34 + \frac{1}{x^2}}$

(3) $f(x) = \frac{x^3 - 4x^2 + \sqrt{x}}{x}$

(4) $f(x) = \frac{2x^3 - \sqrt[3]{x^2}}{2}$

(5) $f(x) = \sqrt{2x} + \sqrt{5}x$

(6) $f(x) = 3x^2 - \pi^2$

1.3 Exponential Functions

You may recall e being vaguely defined in Precalculus: $e \approx 2.718....$ The definition of e involves **limits**, so we are ready for it now!

Definition 1.2

$$e \text{ is the number such that } \lim_{h\to 0} \frac{e^h - 1}{h} = 1$$

Definition 1.3 (Definition of Derivative of $y = x^x$)

$$\frac{d}{dx} e^x = e^x$$

Example 1.4 *Compute the derivative of the function* $f(x) = 2e^x - 3x^2 + 54e$

Example 1.5 *At what point on the curve* $y = 1 + 2e^x - 3x$ *is the tangent line parallel to the line* $3x - y = 5$*?*