

02.07 Derivative as a Function

Sunday, September 13, 2020 1:34 PM



Lecture 8: Section 2.7: The Derivative as a Function

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Today's Goal:

Logistics: Evening quiz this ~~Tuesday~~ ^{today!} Set an alarm on your phone. Speaking of phones...let's try to focus as much as possible during lecture. Take a minute - physically take your phone to the opposite corner of the room (if you are able), and turn on your camera (if you are able)! It will help you say present.

- Drop-back section of pre-calc. starts 9/21 (6:30-7:40pm MTWTh)

Warm-Up 8.1 What is the slope of the tangent line to $f(x) = x^2 + 1$ at the point where $x = 1$? HINT:

$f'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$ is the definition of the slope of the tangent line/instantaneous velocity of a function.

8.1 Definition and Alternate Definition of Derivative

these limits are #'s, b/c "a" is a #

$$f'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$$

$$f'(a) = \lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a}$$

What about $f'(x)$?

x is a variable

if we look at: $\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$

I get a function where I can plug in diff. values into "x" and get tangent line slopes at diff. pts!

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{(x+h)^2 + 1 - (x^2 + 1)}{h} = \lim_{h \rightarrow 0} \frac{x^2 + 2xh + h^2 + 1 - x^2 - 1}{h} = \lim_{h \rightarrow 0} \frac{2xh + h^2}{h}$$

8.2 Position and Velocity

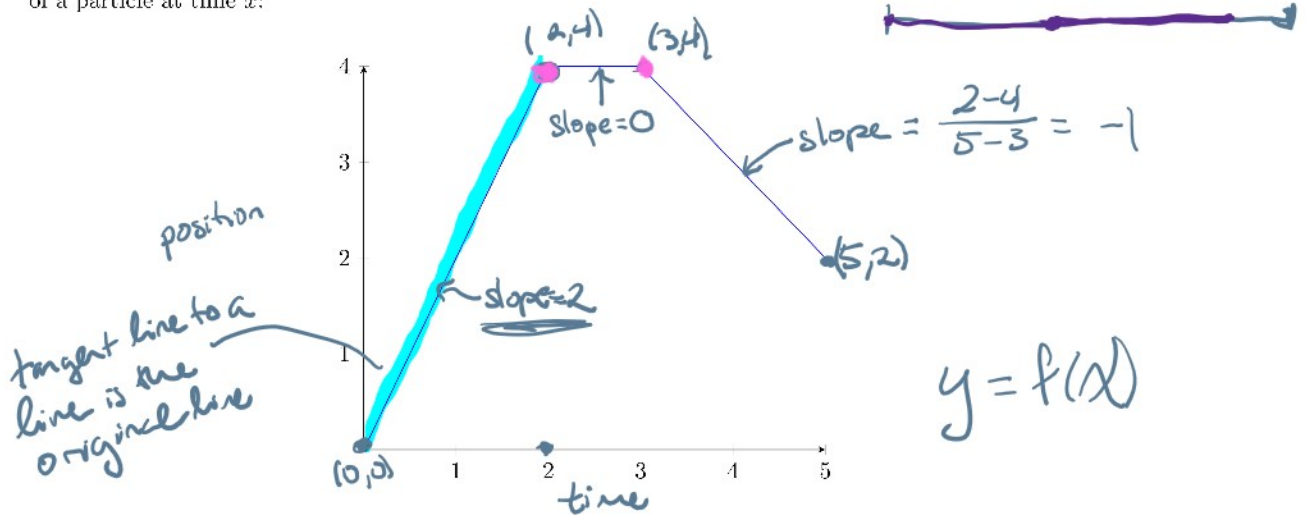
$$= \lim_{h \rightarrow 0} (2x + h) = 2x$$

Suppose $s(t)$ gives the position of a particle at time t .

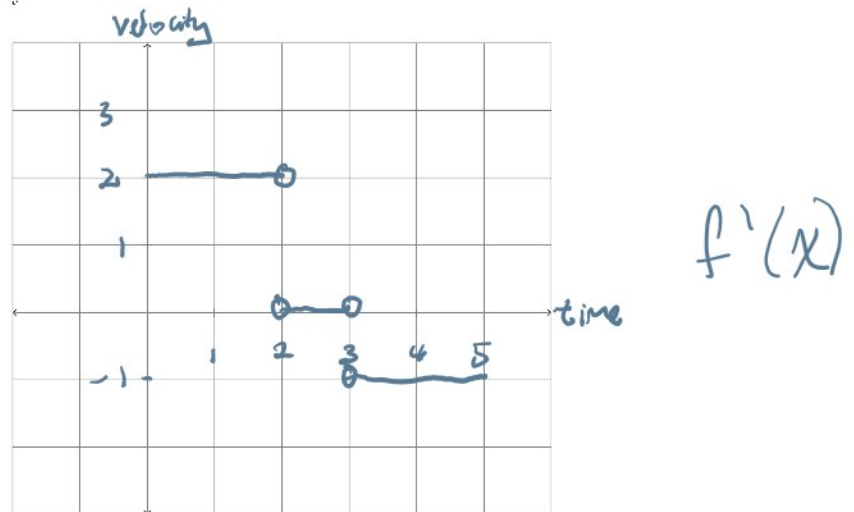
- $\frac{s(b) - s(a)}{b - a}$ = slope of secant line connecting the point $(a, s(a))$ and $(b, s(b))$ = average velocity of particle between times $t = a$ and $t = b$.

- $\lim_{h \rightarrow 0} \frac{s(a+h) - s(a)}{h}$ = slope of tangent line to $s(t)$ at $t = a$ = instantaneous rate of change of particle at $t = a$.
- $s'(a)$

Sometimes we can read this information off of a graph. The following graph depicts the position $y = f(x)$ of a particle at time x :



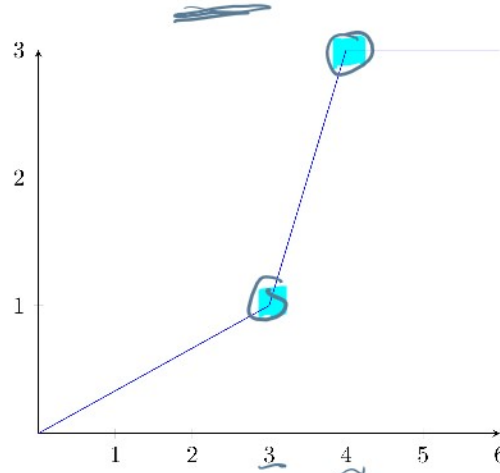
Now, let's graph the velocity:



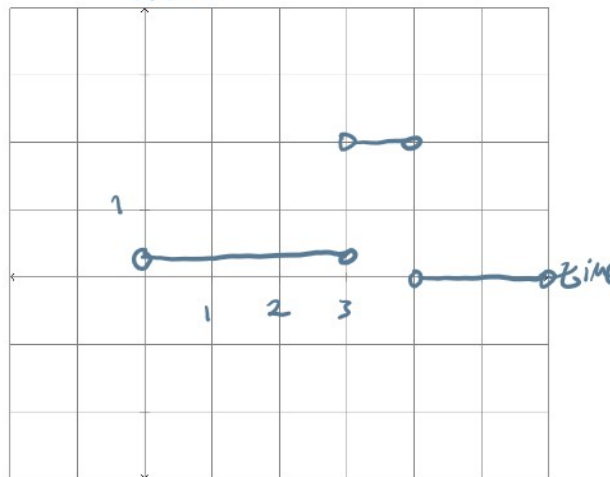
The graph we have drawn depicts the derivative of $y = f(x)$ as a function. We denote this function:

$$f'(x) \text{ or } \frac{dy}{dx}$$

Example 8.2 Suppose this graph depicts the position $y = f(x)$ of some particle at time x :



Now, let's graph the velocity: $f'(x)$ velocity



Example 8.3 Does the expression

$$\lim_{h \rightarrow 0} \frac{e^{x+h} - e^x}{h}$$

$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$
 $f(x) = e^x, f(x+h) = e^{x+h}$
 this is $f'(x)$ for $f(x) = e^x$

represent the derivative of some function? If so, what function?

Example 8.4 Does the expression

$$\lim_{h \rightarrow 0} \frac{\sin(h) - \sin(0)}{h}$$

No x 's, so this is not likely to be " $f'(x)$ "

represent the derivative of some function? If so, what function?

So this is $f'(a)$ for $f(x) = \sin(x)$

But: $f'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$

if $f(x) = \sin x$, then $f(0) = \sin(0)$, $f(0+h) = \sin(h)$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$f''(x)$ is the derivative of $f'(x)$

8.3 Higher Derivatives

If the derivative of $f(x)$ is itself a function, $f'(x)$, then we can continue taking derivative of $f'(x)$!

If $s(t)$ is position and $s'(t) = v(t)$ is velocity, what might $s''(t) = v'(t)$ represent?

$s''(t) = a(t)$ is the acceleration at time t

$s'''(t) = \text{"jerk"}$

Other notation of the second derivative:

$$f''(x) \sim f'(x)$$

$$\frac{d^2y}{dx^2} \sim \frac{dy}{dx}$$

And higher derivatives:

$f'''(x), f^{(4)}(x)$, Not to be confused w/ $f^4(x)$

$$\frac{d^3y}{dx^3}, \frac{d^4y}{dx^4}, \dots$$

8.4 Can we always take a derivative?

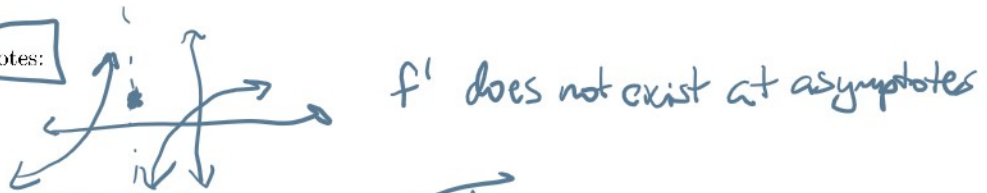
No, not all functions have tangent lines at every point! Can you think of a function with a point where a tangent line might not be a well defined notion?



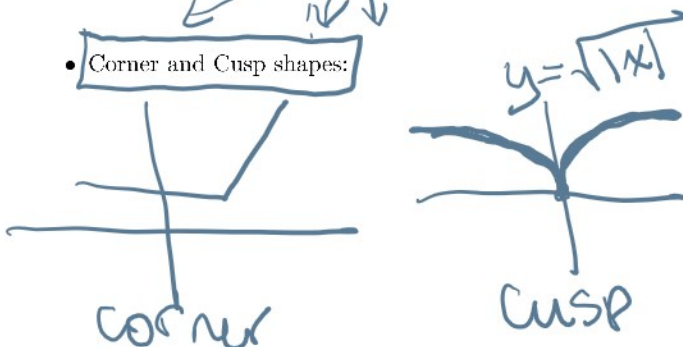
• Continuity:

If $f(x)$ is discontinuous at $x=c$, then $f'(c)$ is not defined. Discontinuities: Jump, hole, infinite

• Asymptotes:



• Corner and Cusp shapes:



f' is not defined at x -values of cusps or corners.