02.07 Derivative as a Function

Sunday, September 13, 2020 1:34 PM

Math 1300: Calculus I **Fall 2020**

Lecture 8: Section 2.7: The Derivative as a Function

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Today's Goal:

Logistics: Evening quiz this Tuesday! Set an alarm on your phone.

Speaking of phones...let's try to focus as much as possible during lecture. Take a minute - physically take your phone to the opposite corner of the room (if you are able), and turn on your camera (if you are able)! It will help you say present.

– Drop-back section of pre-calc. starts 9/21 (6:30 – 7:40pm MTWTh)

Warm-Up 8.1 *What is the slope of the tangent line to $f(x) = x^2 + 1$ at the point where $x = 1$? HINT: $f'(a) = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$ is the definition of the slope of the tangent line/instantaneous velocity of a function.*

$f'(1) = \lim_{h\to 0} \frac{f(1+h)-f(1)}{h} = \lim_{h\to 0} \frac{(1+h)^2+1-(1^2+1)}{h} = \lim_{h\to 0} \frac{1+2h+h^2+1-2}{h} = \lim_{h\to 0} \frac{2h+h^2}{h} = \lim_{h\to 0} \frac{\cancel{h}(2+h)}{\cancel{h}} = \boxed{2}$

8.1 Definition and Alternate Definition of Derivative

these limits are #'s, b/c "a" is a #

$$\boxed{f'(a) = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h}}$$

$$\boxed{f'(a) = \lim_{x\to a} \frac{f(x)-f(a)}{x-a}}$$

What about $f'(x)$?

* x is a variable

if we look at: $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$,

I get a function where I can plug in diff. values into "x" and get tangent line slopes at diff. pts!

$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{(x+h)^2+1-(x^2+1)}{h} = \lim_{h\to 0} \frac{\cancel{x^2}+2xh+h^2\cancel{+1}\cancel{-x^2}\cancel{-1}}{h} = \lim_{h\to 0} \frac{2xh+h^2}{h}$

$= \lim_{h\to 0} (2x+h) = \boxed{2x}$

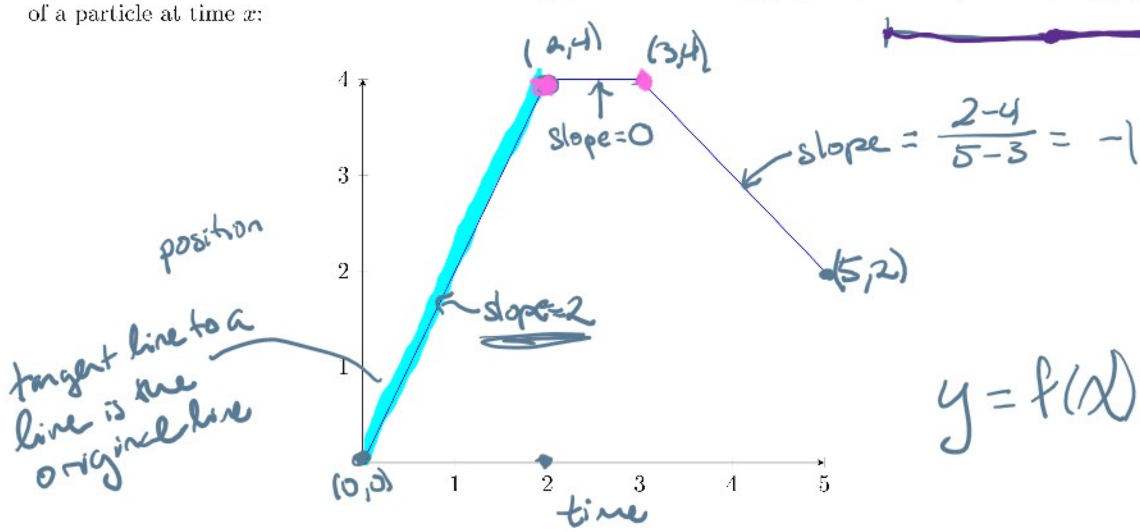
8.2 Position and Velocity

Suppose $s(t)$ gives the position of a particle at time t.

- $\frac{s(b)-s(a)}{b-a}$ = slope of secant line connecting the point $(a, s(a))$ and $(b, s(b))$ = average velocity of particle between times $t = a$ and $t = b$.
- $\lim_{h\to 0} \frac{s(a+h)-s(a)}{h}$ = slope of tangent line to $s(t)$ at $t = a$ = instantaneous rate of change of particle at $t = a$.

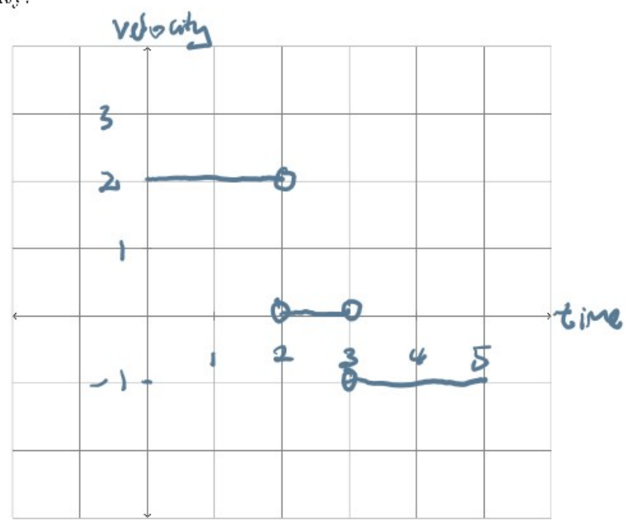
$s'(a)$

Sometimes we can read this information off of a graph. The following graph depicts the position $y = f(x)$ of a particle at time x:



$y = f(x)$

Now, let's graph the velocity:

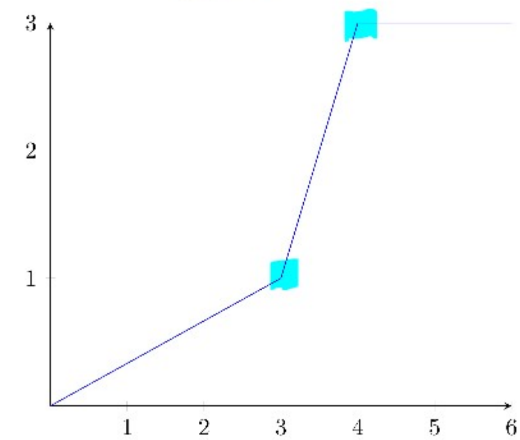


$f'(x)$

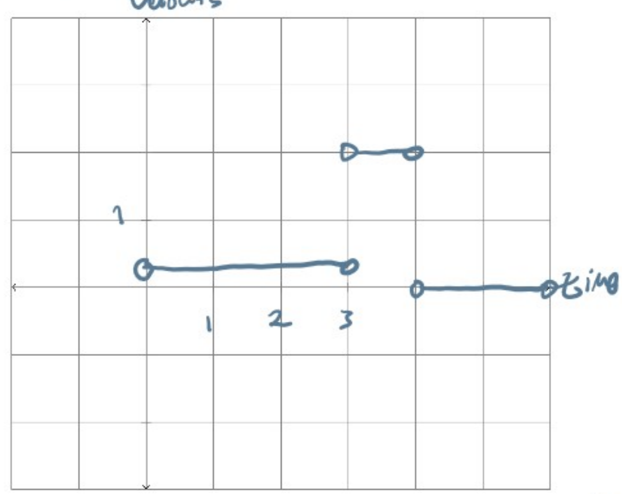
The graph we have drawn depicts the derivative of $y = f(x)$ as a funciton. We denote this function:

$$f'(x) \text{ or } \frac{dy}{dx}$$

Example 8.2 *Suppose this graph depicts the position $y = f(x)$ of some particle at time x:*



Now, let's graph the velocity: $f'(x)$
velocity



Example 8.3 *Does the expression*

$$\lim_{h\to 0} \frac{e^{x+h} - e^x}{h}$$

represent the derivative of some function? If so, what function?

$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$

$f(x) = e^x$, $f(x+h) = e^{x+h}$

this is $f'(x)$ for $f(x) = e^x$

Example 8.4 *Does the expression*

$$\lim_{h\to 0} \frac{\sin(h) - \sin(0)}{h}$$

represent the derivative of some function? If so, what funciton?

No x's, so this is not likely to be "$f'(x)$".

But: $f'(a) = \lim_{h\to 0} \frac{f(a+h) - f(a)}{h}$

If $f(x) = \sin x$, then $f(0) = \sin(0)$, $f(0+h) = \sin(h)$

so this is $f'(0)$ for $f(x) = \sin(x)$

8.3 Higher Derivatives

$f''(x)$ is the derivative of $f'(x)$

If the derivative of $f(x)$ is itself a function, $f'(x)$, then we can continue taking derivative of $f'(x)$!

If $s(t)$ is position and $s'(t) = v(t)$ is velocity, what might $s''(t) = v'(t)$ represent?

$s''(t) = a(t)$ is the acceleration at time t

$s'''(t) =$ "jerk"

Other notation of the second derivative:

$f''(x) \sim f'(x)$

$\frac{d^2y}{dx^2} \sim \frac{dy}{dx}$

And higher derivatives:

$f'''(x)$, $\underbrace{f^{(4)}(x)}$, Not to be confused w/ $f^4(x)$

$\frac{d^3y}{dx^3}$, $\frac{d^4y}{dx^4}$, ...

8.4 Can we always take a derivative?

No, not all functions have tangent lines at every point! Can you think of a function with a point where a tangent line might not be a well defined notion?

- Continuity:

- Asymptotes:

- Corner and Cusp shapes: