02.06 Derivatives and Rates of Change

Wednesday, September 9, 2020 11:56 AM

Math 1300: Calculus I **Fall 2020**

Lecture 7: Section 2.6: Derivatives and Rates of Change

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Today's Goal:

Logistics: We have a check-in today!

We have Quiz 2 Tuesday night. It will cover 2.4 - 2.6 and Project 3 (in general, quizzes cover everything from the what the last quiz covered up until what we cover the Monday before the quiz, but not this time since we will only half-finish 2.7 on Monday).

Warm-Up 7.1 *Evaluate the infinite limit:*

$$\boxed{\lim_{x\to -1^-} \frac{x+1}{x^2-1}}$$

$$\lim_{x\to -1^-}\left(\frac{1(x+1)}{(x+1)(x-1)}\right) = \lim_{x\to -1^-}\left(\frac{1}{x-1}\right)$$

Hole @ $x = -1$

(VA: $x = 1$)

-1 is in domain of this cancelled expression

$$= \frac{1}{-1-1} = \frac{1}{-2}$$

7.1 Slopes of Secant Lines and Tangent Lines

Red: Tangent lines

Blue: Secant lines

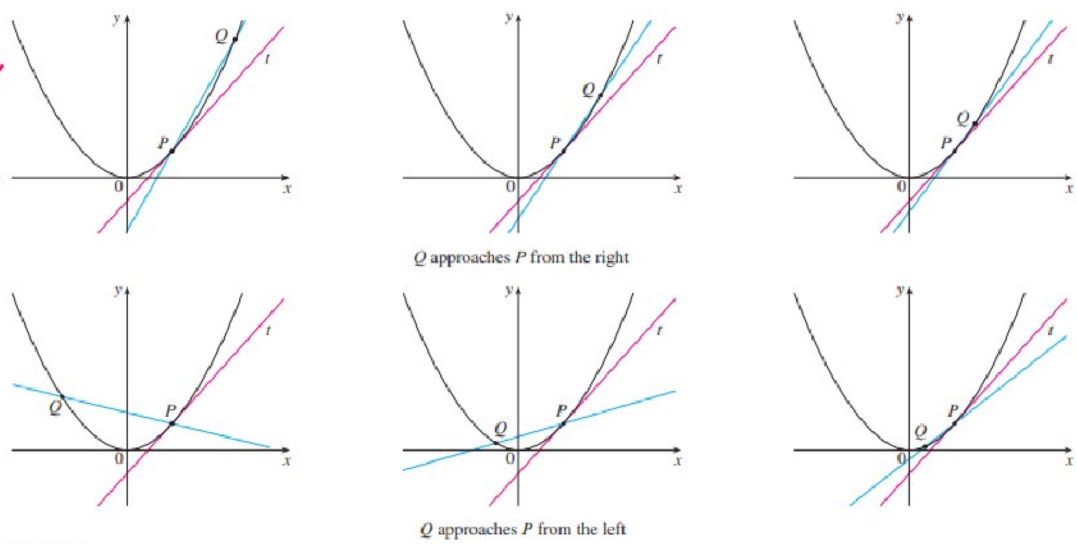


FIGURE 3

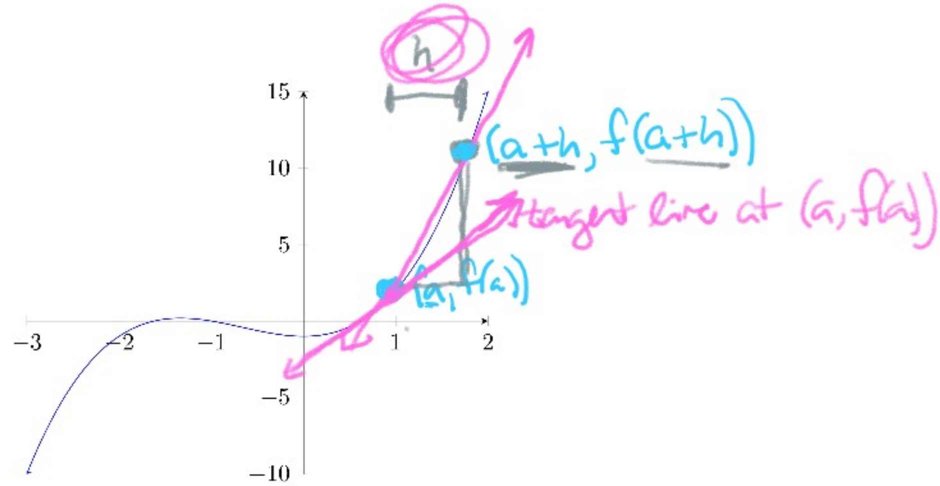
Suppose $P = (2, 1)$ and $Q = (3, 4)$ are two points on $f(x) = x^2 - 2x + 1$.

- Slope of the line between P and Q: $m = \frac{\Delta y}{\Delta x} = \frac{4-1}{3-2} = \boxed{3}$

- Slope of the tangent line at P:

$m = \frac{f(b)-f(a)}{b-a}$ is the slope of line containing $(a, f(a)), (b, f(b))$

7.2 In general



- The slope of the secant line connecting the points $(a, f(a))$ and $(a + h, f(a + h))$ is:

$$m = \frac{f(a+h)-f(a)}{(a+h)-a} = \boxed{\frac{f(a+h)-f(a)}{h}}$$

slope of sec. line

The difference quotient gives the average rate of change.

slope of tangent line to graph @ (a, f(a))

- To find the exact slope of the tangent line, take a limit:

$$\boxed{f'(a)} = \lim_{h\to 0}\left[\text{slope of sec line}\right] = \boxed{\lim_{h\to 0}\frac{f(a+h)-f(a)}{h}}$$

← Notation

The derivative at a point gives the instantaneous rate of change. We can obtain this as a single value, $f'(a)$, or evaluate the limit with x as a variable to obtain a function $f'(x)$ which gives the instantaneous rate of change of f for any x we want to plug in.

f'(a) is a #

f'(x) is a function of the variable x.

line eq: $y = m(x - x_1) + y_1$ (2, −6)

Example 7.2 *Find the equation of the tangent line to $f(x) = 3x - 3x^2$ at $x = 2$.*

$x_1 = 2$
$y_1 = f(2) = 6 - 3\cdot 4 = -6$

$m = \underbrace{f'(2)}_{\text{slope of tangent line @ } x=2} = \lim_{h\to 0} \frac{[f(2+h)] - f(2)}{h}$

$= \lim_{h\to 0} \frac{[3(2+h) - 3(2+h)^2] - -6}{h}$

$= \lim_{h\to 0} \frac{6 + 3h - 3(4 + 4h + h^2) + 6}{h}$

$= \lim_{h\to 0} \frac{6 + 3h - 12 - 12h - 3h^2 + 6}{h}$

$= \lim_{h\to 0} \frac{-3h^2 - 9h}{h}$

$= \lim_{h\to 0} \frac{h(-3h - 9)}{h}$

$= \lim_{h\to 0} (-3h - 9) = \boxed{-9}$ slope of tangent line!

ANS $\boxed{y = -9(x-2) - 6}$ tangent line eq.

Example 7.3 *If $s(t) = 3t - t^2$ gives the position in feet as a function of time t in minutes, what is the instantaneous velocity at $t = 2$?*

slope of tangent line of position function.

$s'(2) = \lim_{h\to 0} \frac{s(2+h) - s(2)}{h}$

$= \lim_{h\to 0} \frac{3(2+h) - (2+h)^2 - [3(2) - 2^2]}{h}$

$= \lim_{h\to 0} \frac{6 + 3h - (4 + 4h + h^2) - [2]}{h}$

$= \lim_{h\to 0} \frac{6 + 3h - 4 - 4h - h^2 - 2}{h}$

$= \lim_{h\to 0} \frac{-h^2 - h}{h}$

$= \lim_{h\to 0} \frac{h(-h-1)}{h}$

$= \lim_{h\to 0} (-h - 1) = -1$ $\boxed{-1 \text{ ft/min}}$

Example 7.4 *What is the equation of the tangent line to $f(x) = \sqrt{x-1}$ at $x = 2$?*

Need point: $x = 2$, $y = f(2) = \sqrt{2-1} = 1$

Need slope: $m = f'(2)$:

$f'(2) = \lim_{h\to 0} \frac{f(2+h) - f(2)}{h}$

$= \lim_{h\to 0} \frac{\sqrt{2+h-1} - 1}{h}$

$= \lim_{h\to 0} \frac{\sqrt{1+h} - 1}{h}$ Rationalize numerator!

$= \lim_{h\to 0} \frac{(\sqrt{1+h} - 1)}{h} \cdot \frac{(\sqrt{1+h} + 1)}{(\sqrt{1+h} + 1)}$

$= \lim_{h\to 0} \frac{(1+h) - \sqrt{1+h} + \sqrt{1+h} - 1}{h(\sqrt{1+h} + 1)}$

$= \lim_{h\to 0} \frac{\cancel{h}}{\cancel{h}(\sqrt{1+h} + 1)}$

$= \lim_{h\to 0} \frac{1}{\sqrt{1+h} + 1} = \frac{1}{2}$

Tangent line: $y = \frac{1}{2}(x-2) + 1$

Example 7.5 *What is the equation of the tangent line to $f(x) = \frac{3}{2x}$ at $x = 3$?*

Need point: $x = 3$, $y = f(3) = \frac{1}{2}$

Need slope: $f'(3)$:

$f'(3) = \lim_{h\to 0} \frac{f(3+h) - f(3)}{h}$

$= \lim_{h\to 0} \frac{\frac{3}{2(3+h)} - \frac{1}{2}}{h} \cdot \frac{\left(\frac{2(3+h)}{1}\right)}{\left(\frac{2(3+h)}{1}\right)}$ clear denom.'s

$= \lim_{h\to 0} \frac{3 - (3+h)}{h(2(3+h))}$

$= \lim_{h\to 0} \frac{-h}{h(2(3+h))}$

$= \lim_{h\to 0} \frac{-1}{2(3+h)} = -\frac{1}{6}$

Tangent line: $y = -\frac{1}{6}(x-3) + \frac{1}{2}$