02.05 Limits Involving Infinity

Tuesday, September 8, 2020 9:36 AM

Math 1300: Calculus I **Fall 2020**

Lecture 6: Section 2.5: Limits Involving Infinity

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Today's Goal: To describe limits that go to infinity, and limits as x goes to infinity.

Logistics: Check-in 4 will cover continuity, discontinuities, and limits involving infinity.

Check-in 5 will cover limits involving infinity

Check out the parent functions pdf in "Weekly work" on Canvas

please email me when you send me a question on webassign

$x^2 = -3$, not a concern, no real # squares to -3

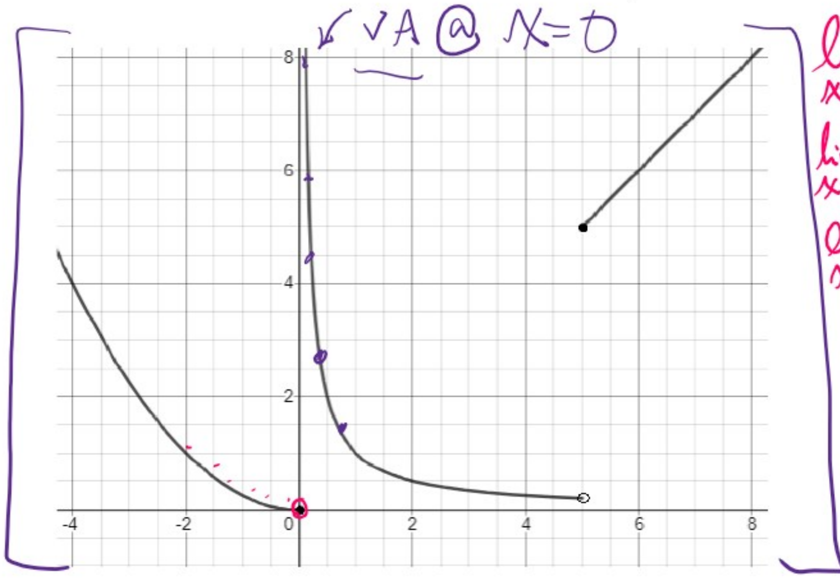
Warm-Up 6.1 $\boxed{\textit{True}}$ *or False:* $f(x) = \frac{x-1}{\sqrt{x^2+3}}$ *is continuous on* $(-\infty, \infty)$.

$x^2+3 > 0$ on

• denominators $\neq 0$ & radicals: no neg. under radical!

6.1 Infinite Limits: Vertical Asymptotes

We have had some informal discussion about this before. Recall this example from our Section 2.2 Notes:



$\lim_{x\to 0^+} f(x) = +\infty$

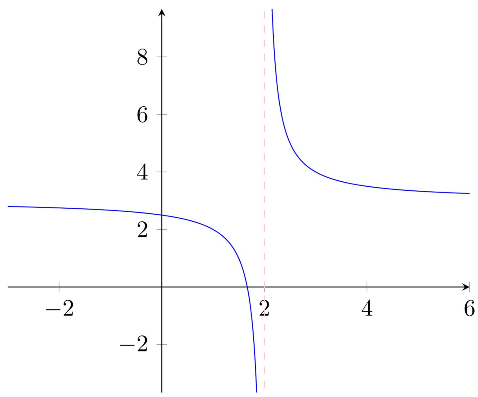
$\lim_{x\to 0} f(x) = DNE$

$\lim_{x\to 0^-} f(x) = 0$

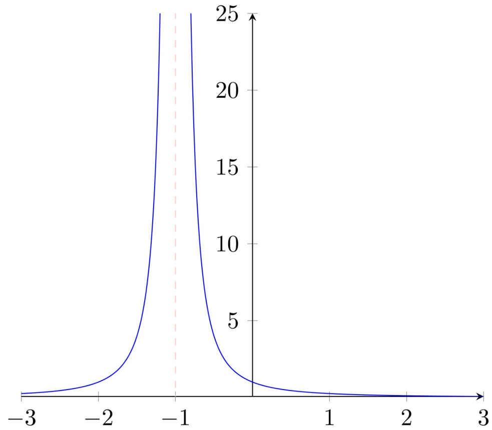
We discussed $\lim_{x\to 0^+} f(x)$ and concluded that this limit **does not exist** (because it's not approaching a real number as we get closer to 0 from the right), but we can do better than that: We can specify that the function values are approaching positive infinity. We do this by writing:

$$\lim_{x\to 0^+} f(x) = \infty.$$

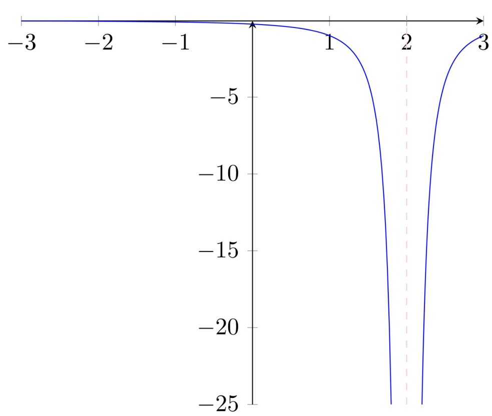
This is an infinite limit. We use $\pm\infty$ and "DNE" to distinguish between different types of vertical asymptotes:



$\lim_{x \to 2} f(x) = DNE$



$\lim_{x \to -1} f(x) = +\infty$



$\lim_{x \to 2} f(x) = -\infty$

6.1.1 Identifying from an equation

Vertical asymptotes happen with a variety of different types of functions:

☆ look up: graphing rational functions notes!!

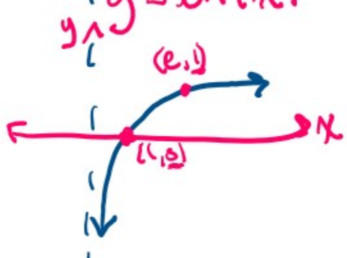
- rational functions (quotient of a polynomial)

$$f(x) = \frac{(x-1)\cancel{(x-2)}}{\boxed{(x+1)}\cancel{(x-2)}} \quad \text{has VA: } \boxed{x=-1}$$

does not cancel - gives us a VA. hole @ $x=2$

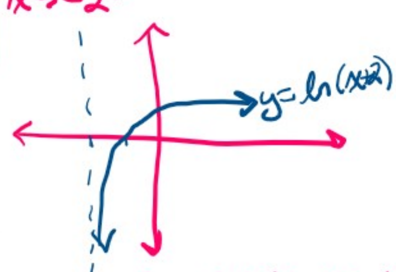
- log functions

$y = \ln(x)$

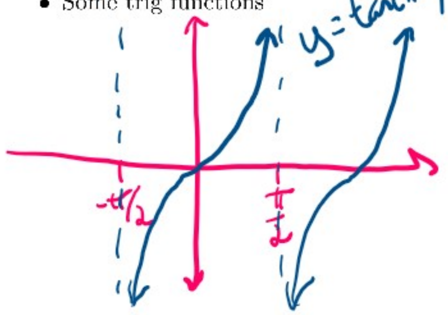


$$\boxed{\lim_{x\to 0^+} \ln(x) = -\infty}$$

$$\lim_{x\to -2^+} \ln(x+2) = -\infty$$



- Some trig functions



$$\lim_{x\to \pi/2^-} \tan(x) = +\infty$$

$$\lim_{x\to \pi/2^+} \tan(x) = -\infty$$

$$\lim_{x\to \pi/2} \tan(x) = \text{DNE}$$

cot(x), sec(x), csc(x) also all have VA's!

Example 6.2 *Evaluate* $\lim_{x\to 0} \ln(\sin(x))$ ⟶ DNE, b/c left vs. right issues!

☆ $\lim_{x\to 0} (\sin(x)) = 0$

$$\lim_{x\to 0} \ln(\underbrace{\sin(x)}_{\to 0})$$

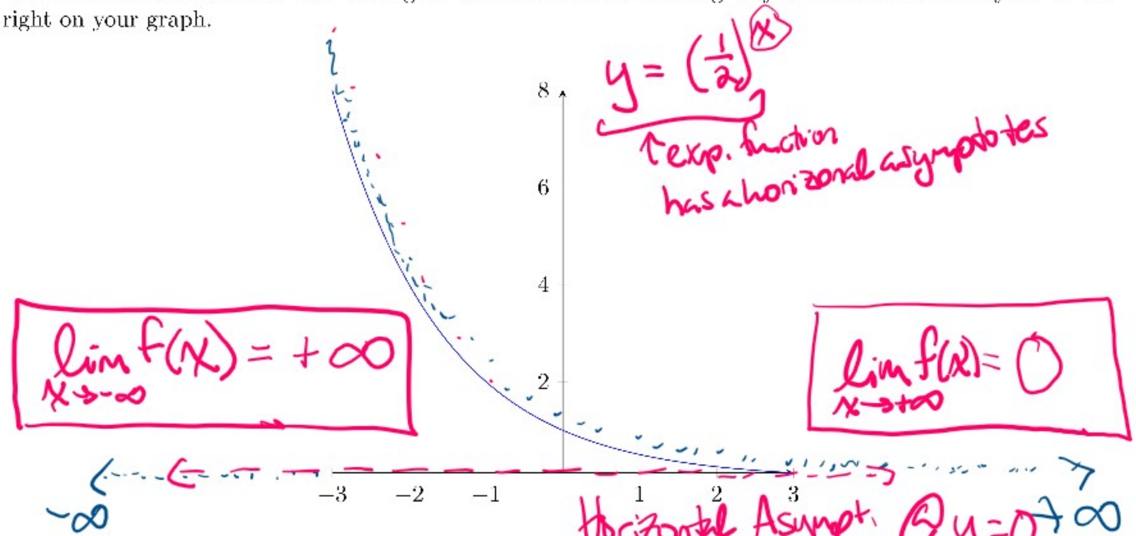
$\lim_{x\to 0^+} \ln(x) = -\infty$

$\lim_{x\to 0^-} \ln(x) = \text{DNE}$ - Out of domain!

fix: $\boxed{\lim_{x\to 0} \ln(|\sin(x)|) = -\infty}$

6.2 Limits as x approaches $\pm$ Infinity

We can also consider limits as x itself goes to $\pm\infty$: Think of walking very far to the left or very far to the right on your graph.



If you don't have the graph to look at, you can plug in very large values of x to see the limit as $x \to \infty$ (like 1000, etc.), or very negative numbers to see the limit as $x \to -\infty$ (like -1000).
... as long as you know what to expect from that type of function

Definition 6.3 *The line $y = L$ (where L is some constant number) of the curve $y = f(x)$ if either*

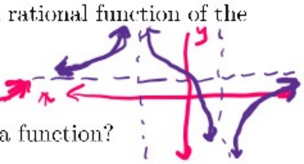
$$\boxed{\lim_{x\to\infty} f(x) = L} \text{ or } \boxed{\lim_{x\to-\infty} f(x) = L}$$

6.3 Pre-calc Techniques

You have experience finding horizontal asymptotes of rational functions! Consider a rational function of the form

★ For rational functions HA's happen to both sides: $x \to +\infty$, $x \to -\infty$

$$\frac{f(x)}{g(x)} \quad \begin{matrix} \sim x^2-3x+4 \ldots \\ \sim 2-x^3 \ldots \end{matrix}$$



where f and g are polynomials. How do you find the horizontal asymptote of such a function?
"degree comparison test"

- If the degree of $f(x)$ is smaller than the degree of $g(x)$,
  $y = \frac{2x}{x^3+1}$ "$\frac{2\cdot\infty}{\infty^3+1}$" ← deg 1, ← deg 3 ⇝ goes to 0. $\boxed{HA: y=0}$
- If the degrees of $f(x)$ and $g(x)$ are the same,
  $y = \frac{3-2x^2}{4x^2+2x+1}$ ⇝ "$\frac{-2\cdot\infty^2}{4\cdot\infty^2}$" ⇝ "$\frac{-2}{4}$" ⇝ goes to $-\frac{1}{2}$, $\boxed{HA: y=-\frac{1}{2}}$
- If the degree of $f(x)$ is larger than the degree of $g(x)$,
  $y = \frac{x^3+1}{2x}$ ⇝ "$\frac{\infty^3}{2\cdot\infty}$" ⇝ goes to $+\infty$ ⇝ $\lim_{x\to+\infty}\left(\frac{x^3+1}{2x}\right) = +\infty$

$\lim_{x\to-\infty}\left(\frac{x^3+1}{2x}\right)$ ⇝ "$\frac{(-\infty)^3}{2(-\infty)}$" ⇝ "$\frac{-\infty^3}{-2\cdot\infty}$" $= \infty$

$\lim_{x\to-\infty}\left(\frac{x^3+1}{2x^2}\right)$ = "$\frac{-\infty^3}{2(-\infty)^2}$" ⇝ $\frac{-\infty^3}{2\cdot(\infty)^2}$ $= -\infty$

Example 6.4 *Find the equation for the horizontal asymptote of the function* $f(x) = \frac{3x^2-2x}{x-2x^2}$

$\lim_{x\to\infty} f(x) = \frac{3}{-2}$

$\lim_{x\to-\infty} f(x) = \frac{3}{-2}$

$\lim_{x\to\infty}\left(\frac{3x^2-2x}{x-2x^2}\right)$ mult. num & denom by: $\frac{1}{x^2}$ ← highest power of x present.

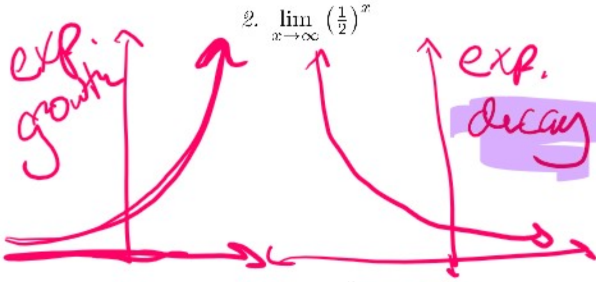
$= \lim_{x\to\infty}\left[\frac{(3x^2-2x)\cdot\frac{1}{x^2}}{(x-2x^2)\frac{1}{x^2}}\right] = \lim_{x\to\infty}\left[\frac{3-2\cdot\frac{1}{x}}{\frac{1}{x}-2}\right] \xrightarrow{*\lim_{x\to\infty}\frac{1}{x}=0} \frac{3-0}{0-2} = \boxed{\frac{3}{-2}}$

Example 6.5 *Evaluate the following limits:*

1. $\lim_{x\to-\infty}\frac{x^2-3}{x^5+1} = 0$

$\lim_{x\to-\infty}\left[\frac{(x^2-3)\left(\frac{1}{x^5}\right)}{(x^5+1)\left(\frac{1}{x^5}\right)}\right] = \lim_{x\to-\infty}\frac{\frac{1}{x^3}-3\frac{1}{x^5}}{1+\frac{1}{x^5}} = \frac{0-0}{1} = \boxed{0}$

2. $\lim_{x\to\infty}\left(\frac{1}{2}\right)^x$



$\frac{1}{2} < 1$, so decay

$\lim_{x\to\infty}\left(\frac{1}{2}\right)^x = 0$

3. $\lim_{x\to-\infty}\frac{2}{x^3} = 0$

4. $\lim_{x\to\infty} e^x$ ⇝ exponential growth, b/c $e > 1$

$= \infty$