02.04 Continuity

Tuesday, September 1, 2020 12:29 PM

Math 1300: Calculus I **Fall 2020**

Lecture 5: Section 2.4: Continuity

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Today's Goal: Learn the definition of a continuous function, and some important theorems of continuo

Logistics:

Check-in Friday, Written Homework due Thursday by 6pm

We are starting this section Wednesday (possibly Tuesday?), but we will not finish it until Friday.

Warm-Up 5.1 *Evaluate* $\lim_{x\to 1} \frac{x-1}{\sqrt{x}-1}$.

$\lim_{x\to 1} \frac{x-1}{\sqrt{x}-1}\cdot\frac{(\sqrt{x}+1)}{(\sqrt{x}+1)} = \lim_{x\to 1} \frac{(x-1)(\sqrt{x}+1)}{x-\sqrt{x}+\sqrt{x}-1} = \lim_{x\to 1}\frac{(x-1)(\sqrt{x}+1)}{(x-1)} = \lim_{x\to 1}(\sqrt{x}+1) = 2$

5.1 Definition

Definition 5.2 *A function $f(x)$ is continuous at a value $x = a$ if*

$$\lim_{x\to a} f(x) = f(a).$$

$f(x)=\sqrt{x}$ is continuous at $x=1$

There is a lot hidden in this definition!

- $\lim_{x\to a} f(x)$ exists! We need the left and right limits to agree for this to be true.
- $f(a)$ needs to exist! a needs to be in the domain of the function for this to happen.
- The above two quantities need to be equal.

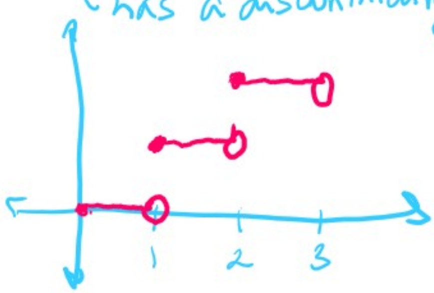
If $f(x)$ does not satisfy this definition at the value a, we say $f(x)$ **is discontinuous at** a, or $f(x)$ **has a discontinuity at** a.

$f(x) = [[x]]$ ~ "rounding down function" — has a discontinuity at any integer

$[[2.1]] = 2$

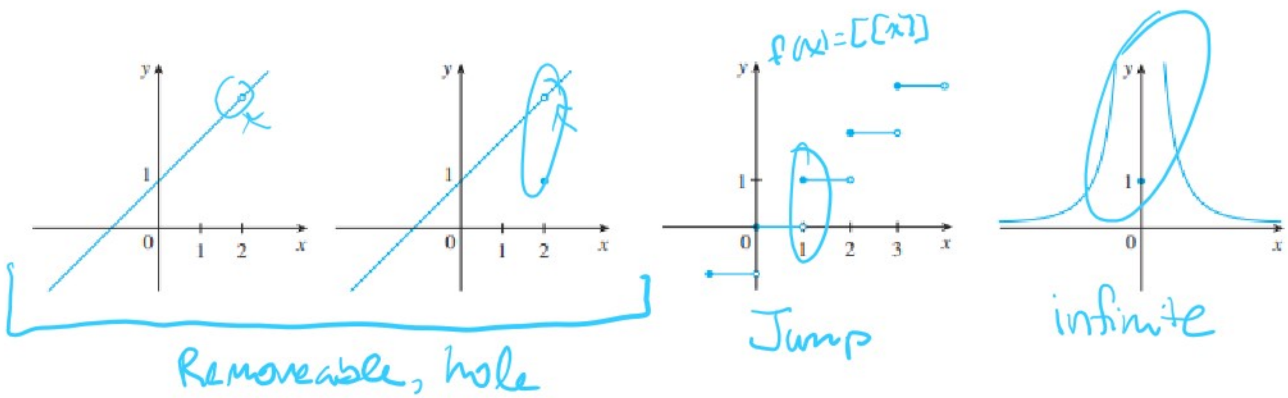
$[[2]] = 2$

$[[1.9]] = 1$



5.2 Discontinuities: From a Graph

Where are the following functions continuous? Where are they discontinuous? Can we classify the types of discontinuities these different graphs display? There are three kinds of discontinuity: Jump, Removeable, and Infinite.



Continuous functions have graphs you can draw without picking up your pencil

5.3 Discontinuities: From an Equation

5.3.1 Domain

This will have a similar feel to finding the domain of a function given by an equation. Recall, you need to check things like **denominators** and **even radicals** (square roots, 4th roots, etc.).

Example 5.3 *Find the domain of the function:*

$$r(t) = \frac{(\sqrt{t+1})(\sqrt[3]{t-6})}{t^2+3t-4}$$

5.3.2 Discontinuities

Example 5.4 *Consider the function:*

$$h(x) = \begin{cases} \frac{x^2-1}{x^3+8} & \text{, if } x \neq -2 \\ 4 & \text{, if } x = -2 \end{cases}$$

Where is $h(x)$ discontinuous? Where is $h(x)$ continuous? Use interval notation to answer.

Example 5.5 *Where is the following function continuous?*

$$h(t) = \frac{1}{x}$$

Example 5.6 *Where is the following function continuous?*

$$f(x) = \begin{cases} \frac{x^2-x-6}{x+2} & \text{, if } x \neq -2 \\ -5 & \text{, if } x = -2 \end{cases}$$

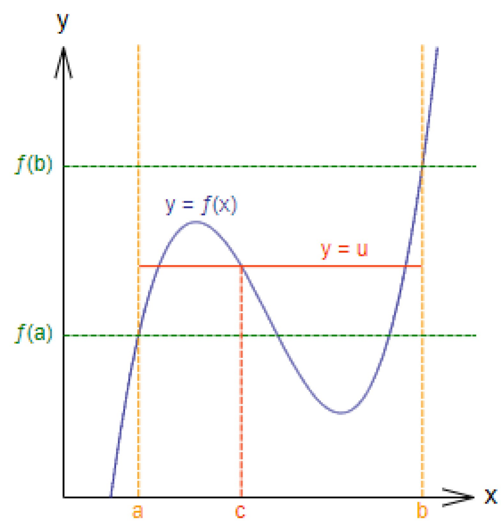
Definition 5.7 *We say that a function is* ***continuous on the left at a*** *if* $\lim_{x \to a^-} f(x) = f(a)$. *We likewise define being continuous on the right.*

Example 5.8 *Is* $[[x]]$ *continuous on the left and/or right at* $a = 5$*?*

Theorem 5.9 *The following types of functions are continuous at every value in their domain:*

- *polynomials*
- *rational functions*
- *root functions*
- *trig functions*
- *exponential functions*
- *log functions*

Theorem 5.10 (The Intermediate Value Theorem) *Suppose f is continuous on the closed interval $[a, b]$ (of x-values). If u is any number between $f(a)$ and $f(b)$, then there exists an x-value c in (a, b) such that $f(c) = u$.*



(Image from Kpengboy / Public domain on the Wikipedia article for Intermediate Value Theorem)

Example 5.11 *Use the intermediate value theorem to show that the function $f(x)$ as below has a root between $x = 0$ and $x = 1$:*

$$f(x) = \sqrt[3]{x} + x - 1$$

Example 5.12 *Use the intermediate value theorem to show that the following equation has a solution in the interval* $(1, 2)$.

$$\sin(x) = x^2 - x$$