Math 1300: Calculus I **Fall 2020**

Lecture 3: Section 2.2: The limit of a function

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Today's Goal: Define the limit of a function, graphically and algebraically

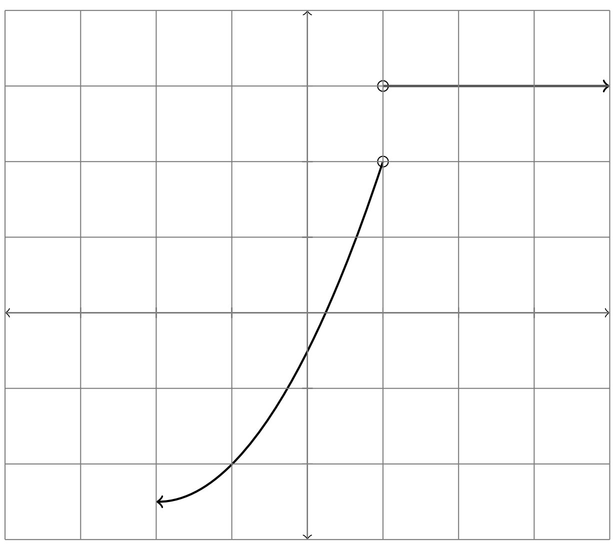
Logistics:

The first Written Homework is due Thursday at 5pm - it is not on new material, but rather a pre-calculus review.

Check-in 1 is today! I'll stop lecture 10 minutes before the end of class and we will all go into Canvas and do the Check-in. Stay on Zoom, cameras on please.

Quiz 1 is next week! You'll need the Proctorio extension in your *Chrome* browser. You'll have practice for this on Thursday, but you can install it now. You can leave it "disabled" until you need it for the Quiz or practice on Thursday.

3.1 Infinitely Close: Visually



To define **the limit of** $f(x)$ **as** x **approaches** a, in your mind, replace the point $(a, f(a))$ with an open circle. What height does it *look like* f is going to reach at a?

Notation:

- Limit of $f(x)$ as x approaches a from the left: $\lim_{x\to a^-} f(x)$
- Limit of $f(x)$ as x approaches a from the right: $\lim_{x\to a^+} f(x)$
- Limit of $f(x)$ as x approaches a: $\lim_{x\to a} f(x)$
  **This one only exists when left and right limits agree!*

Definition 3.1 (The Limit of $f(x)$ as x approaches a from the left) *We write* $\lim_{x \to a^-} f(x) = L$ *and say the left-hand limit of* $f(x)$ *as* x *approaches* a *(or the limit of* $f(x)$ *as* x *approaches* a *from the left) is equal to* L *if we find values of* $f(x)$ *arbitrarily close to* L *by taking* x *to be sufficiently close to* a*, and* $x < a$*.*

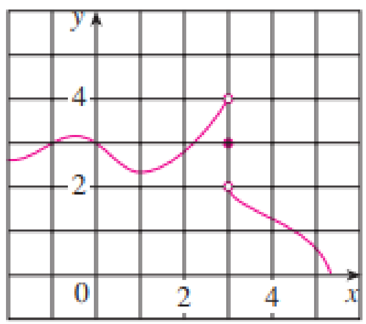
Definition 3.2 (The Limit of $f(x)$ as x approaches a from the right) *We write* $\lim_{x \to a^+} f(x) = L$ *and say the right-hand limit of* $f(x)$ *as* x *approaches* a *(or the limit of* $f(x)$ *as* x *approaches* a *from the right) is equal to* L *if we find values of* $f(x)$ *arbitrarily close to* L *by taking* x *to be sufficiently close to* a*, and* $x > a$*.*

Definition 3.3 (The Limit of $f(x)$ as x approaches a) *We write* $\lim_{x \to a} f(x) = L$ *and say the limit of* $f(x)$ *as* x *approaches* a *is equal to* L *if we find values of* $f(x)$ *arbitrarily close to* L *by taking* x *to be sufficiently close to* a *(on both sides), but with* $x \neq a$*.*

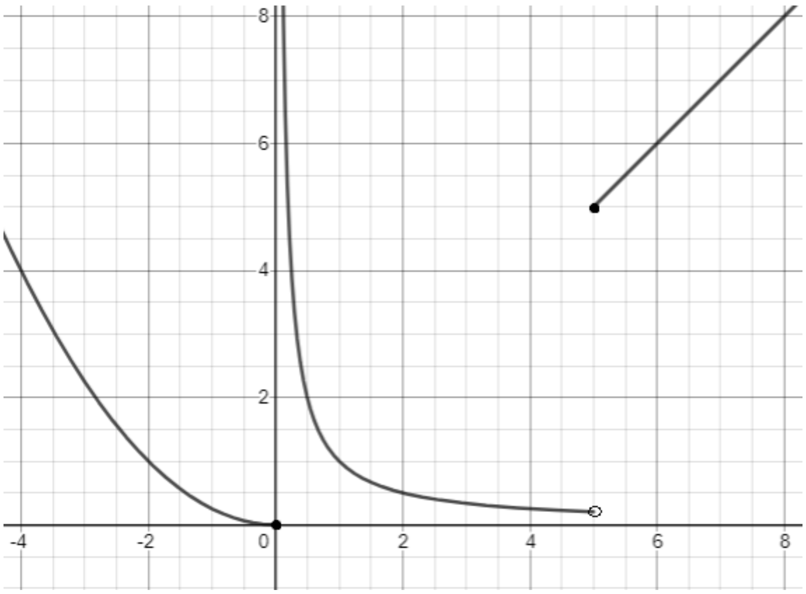
4. For the function f whose graph is given, state the value of each quantity, if it exists. If it does not exist, explain why.

(a) $\lim_{x \to 0} f(x)$ (b) $\lim_{x \to 3^-} f(x)$ (c) $\lim_{x \to 3^+} f(x)$

(d) $\lim_{x \to 3} f(x)$ (e) $f(3)$



Example 3.4



Example 3.5
Find the following values, if they exist:

1. $\lim_{x\to -2} f(x)$
2. $\lim_{x\to 0^-} f(x)$
3. $\lim_{x\to 0^+} f(x)$
4. $f(5)$
5. $f(0)$
6. $\lim_{x\to 0}$
7. $\lim_{x\to 5^+} f(x)$

3.2 Infinitely Close: Numerically

For "reasonable" functions, you can make a table of values to see what the limit might be. This is easiest with a calculator:

Example 3.6 *Find the values of* $f(x) = \frac{x^2-x-2}{x-2}$ *at* $x = 1.9, 1.99, 1.999$ *and take a guess at the value of* $\lim_{x\to 2^-} f(x)$*:*

x	$f(x) = \frac{3x-1}{x-2}$
1.9	
1.99	
1.999	

Find three values of $f(x) = \frac{x^2-x-2}{x-2}$ *and use them to estimate* $\lim_{x\to 2^+} f(x)$.

Does $\lim_{x\to 2} f(x)$ *exist? What do you think the value is?*

Is there a way to see this algebraically? We will learn more algebraic techniques and summarize them when we move to 2.3!

Example 3.7 *What is* $\lim_{x \to 0} \sin\left(\frac{\pi}{x}\right)$*? Try graphing the function in Desmos, plugging in values, etc. Hint: If you plug in fractions, you can actually make your life easier! Recall:* $\frac{1}{1/n} = n$.