02.01 Tangent Lines and Velocity

Tuesday, August 25, 2020 10:51 AM

Math 1300: Calculus I **Fall 2020**

Lecture 2: Section 2.1: Tangent Lines and Velocity and Pre-Calc Review!

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Today's Goal: Tangent Lines Logistics:

The first WebAssign is due tonight!

The first Written Homework is due Thursday at 5pm - it is not on new material, but rather a pre-calculus review.

Check-in 1 is tomorrow! We will do it at the end of class tomorrow. It will open the last 5 minutes of class. The questions are randomized, but they can cover tangent line things (learning today) or pre-calculus review (like today's activity).

Quiz 1 is next week! You'll need the Proctorio extension in your *Chrome* browser. You'll have practice for this on Thursday, but you can install it now. You can leave it "disabled" until you need it for the Quiz or practice on Thursday.

Okay let's start out with Desmos! This activity should take $\sim$ 20 minutes.

$y = e^{x} + e^{-x}$] function, b/c each x gives only 1 output y

even/odd? plug in "$-x$" for x:

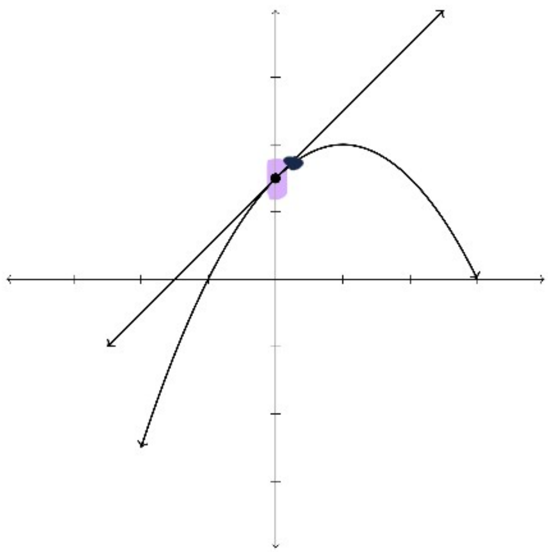
$$y = e^{(-x)} + e^{-(-x)} = e^{-x} + e^{x}$$

Not a func.

2.1 Tangent Lines

2.1.1 Definition

Intuitively, a **tangent line** is a line (in 2D space) that "just touches" a particular point of a curve (also in 2D space). This is a pretty loosey-goosey definition, and the precise definition was difficult to describe without Calculus.



Tangent lines can be useful for measuring steepness of something that is an irregular shape. Steepness can have different meanings, depending on what the axes mean.

Historically, the tangent line was defined by Leibniz as the line through an infinitely close pair of points on the curve. Leibniz and Newton discovered calculus at the same time in the 1600's, independently. With calculus, we will be able to make the statement "infinitely close pair of points" more precise. Not today though.

2.2 Quick Reminders About Lines

We will try to find the equations for tangent lines, but first recall a few things about lines:

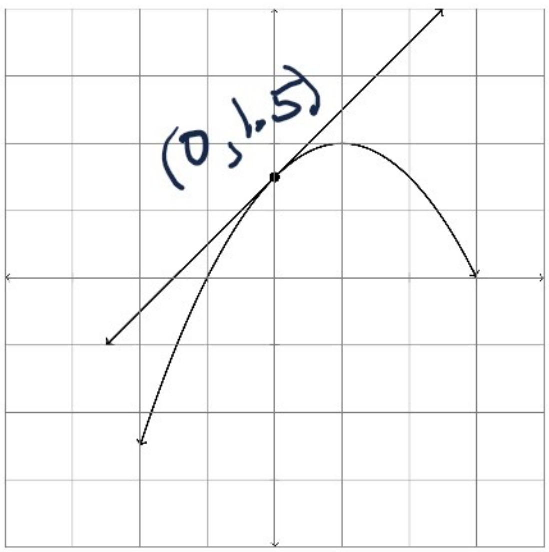
- $y = mx + b$ is the **slope-intercept** form for the equation of a line, where m is the slope of the line and b is the y coordinate of the y-intercept.

  For example: $y = 2x - 3$ has slope 2 and y-intercept $(0, -3)$.

  $m = \frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x}$

- $y = m(x - x_1) + y_1$ is the **point-slope** form for the equation of a line, where m is the slope and (x_1, y_1) is any point on the line.

Example 2.1 (Goal Example For Today) *Find the equation of the tangent line pictured here.*



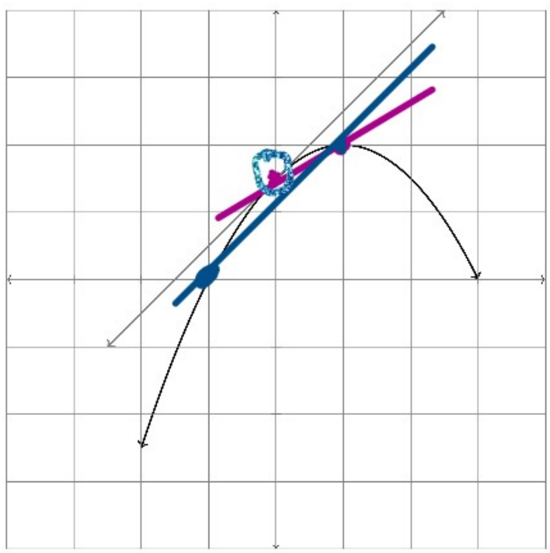
1. What form of the equation of a line should we use?
2. What pieces of information do we need to write that equation? Point: (0, 1.5)

Whichever form we use, finding the **slope** of this line looks a little tricky. We can estimate it using a **secant line**.

2.3 Secant Lines

A good way to approximate a tangent line is to take Leibniz's "infinitely close pair of points" and make it "a pretty darn close pair of points". A **secant line** is a line through a pair of points on a graph.

Let's forget about the tangent line for a second (I greyed it out), and try to find a secant line that is "near" the tangent line. We need to choose two points on this curve. Which two points should we choose? (This part will be drawn in live)



Blue	Pink
slope is a better match!	shares (0, 1.5) with tangent line

Compute the equation for this secant line. $(0,1.5),\ (1,2)$

$$y = m(x - x_1) + y_1$$

$$m = \frac{\Delta y}{\Delta x} = \frac{2-1.5}{1-0} = \frac{1}{2}$$

$$\boxed{y = \frac{1}{2}(x-1) + 2}$$

What could improve our estimate? Bring (1,2) closer to (0,1.5)
(need eq. of parabola to get a pt. w/ x-val. closer to 0)

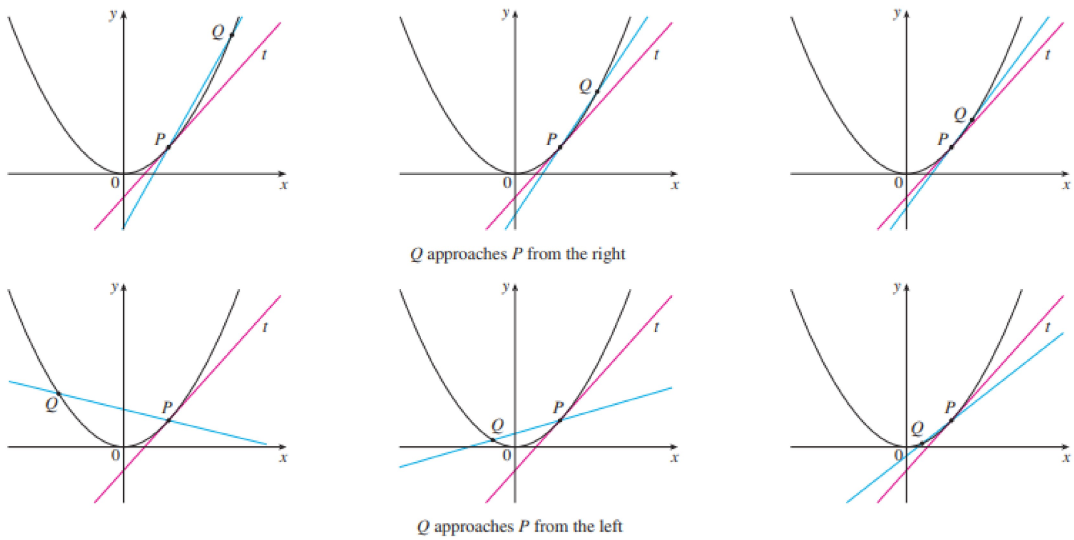
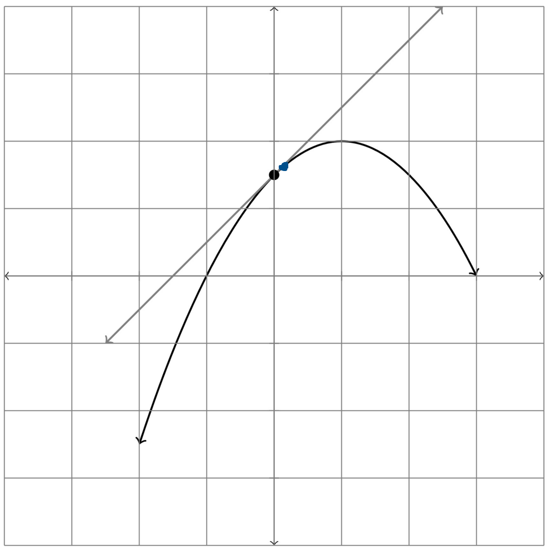


FIGURE 3

Let's try to do better, with the added information that the equation for the curve is $y = \frac{-1}{2}x^2 + x + \frac{3}{2}$:



$(0, 1.5), (.1, 1.595)$

$-\frac{1}{2}(.1)^2 + .1 + 1.5$

$-.5(.01) + 1.6$

$-.005 + 1.6$

1.595

$m = \frac{1.595 - 1.5}{.1 - 0} = \frac{.095}{.1} = .95$

$y = .95(x - 0) + 1.5$

$\boxed{y = .95x + 1.5}$

2.4 Velocity

Velocity gives us an application of the ideas above. There are two types of velocity:

1. average velocity between time t_0 and time $t_1 = \dfrac{\text{change in distance}}{\text{change in time}}$ slope of secant line!
2. instantaneous velocity at a time t_0 is the limiting value of the average velocities as t_1 gets close to t_0

Tangent line slope!

Key Question: How do these concepts (average and instantaneous velocity) relate to tangent and secant lines?

7. The table shows the position of a cyclist.

t (seconds)	0	1	2	3	4	5
s (meters)	0	1.4	5.1	10.7	17.7	25.8

(a) Find the average velocity for each time period:
(i) $[1, 3]$ (ii) $[2, 3]$ (iii) $[3, 5]$ (iv) $[3, 4]$

(b) Use the graph of s as a function of t to estimate the instantaneous velocity when $t = 3$.

Example 2.2 (2.1.7)

$$[3,5]: \frac{25.8-10.7}{5-3} = \frac{15.1}{2} = 7.55 \text{ m/s}$$

use avg. vel. to est. vel at $t=3$

$$[3,4]: \frac{17.7-10.7}{4-3} = \frac{7}{1} = \boxed{7 \text{ m/s}}$$

$[2,3]$: ...