Integrating trigonometric functions

Here are the most basic trigonometric identities

$$\begin{aligned}\sin^2 x + \cos^2 x &= 1 \text{ (Pythagorean identity)}\\ \sin(x+y) &= \sin x \cos y + \cos x \sin y\\ \cos(x+y) &= \cos x \cos y - \sin x \sin y.\end{aligned}$$

From these follow

$$\begin{aligned}\tan^2 x + 1 &= \sec^2 x\\ 1 + \cot^2 x &= \csc^2 x\\ \sin(2x) &= 2\sin x \cos x\\ \cos(2x) &= \cos^2 x - \sin^2 x.\end{aligned}$$

and

$$\cos^2 x = \frac{1+\cos(2x)}{2},\ \sin^2 x = \frac{1-\cos(2x)}{2}.$$

We can use these along with our other integration techniques to integrate new functions. [Appendix C of the text contains a review of trigonometry.] You should also memorize the following:

$$\int \sec x\ dx = \ln|\sec x + \tan x|,\ \int \csc x\ dx = -\ln|\csc x + \cot x|.$$

1. Integrate $\int \sin x \cos x\ dx$ two ways, with a substitution and with a trig identity. Use trig identities to show that your answers are equivalent (i.e. differ by a constant).

2. $\int \cos^5 x \sin^4 x \; dx$

3. $\int \sin^3 x \; dx$

4. $\int \sec^4 x \tan^3 x \; dx$

5. $\int \sec^2 x \tan^3 x \; dx$

6. $\int \cot^3 x \csc^3 x \; dx$

7. $\int \cos^4 x \; dx$

8. $\int \cos^2 x \sin^2 x \; dx$

9. $\int \tan^2 x \; dx$

10. $\int \sec x \; dx$, $\int \csc x \; dx$ (Multiply and divide by $\sec x + \tan x$ or $\csc x + \cot x$.)