MATH 2300-004 QUIZ 8 Name: ____________________________

Collaborators (if any):

Due Monday, April 1st at the beginning of class. Submit your work on additional paper, treating this page as a cover sheet. You may use technology and work with with other students. If you work with others, please list their names above.

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1. Find the interval of convergence for each of the following power series.

   (a) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^n$

   (b) $\displaystyle\sum_{n=0}^{\infty} n^n (x+1)^n$

   (c) $\displaystyle\sum_{n=0}^{\infty} \frac{(-2)^n n}{\sqrt{n^3+1}} (x-1)^n$

2. Starting with the geometric series, $\displaystyle\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$ for $|x| < 1$, show that

$$\pi = 2\sqrt{3} \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)3^n}.$$

   [Hint: Evaluate $\arctan(x)$ at $1/\sqrt{3}$.]

3. For this problem, let $f(x) = (1+x)^{1/3}$

   (a) Find $f'(x)$, $f''(x)$, and $f'''(x)$.

   (b) What is $T_3(x)$, the third degree Taylor polynomial for f centered at $a = 0$?

   (c) Use $T_3(x)$ to estimate $\sqrt[3]{2}$.

4. For this problem, you may assume $\displaystyle e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!}$ (i.e. e^x is equal to its Taylor series centered at zero).

   (a) Integrate term-by-term to evaluate the definite integral

$$\int_0^1 e^{-x^2}\, dx.$$

   [Your answer will be an infinite series.]

   (b) Use the alternating series remainder estimate to to give an approximation to the above integral so that the absolute value of the error is less than 0.001.

5. Find the Taylor series for $\cos x$ centered at $a = \pi/2$ in two ways:

   (a) from the definition (i.e. calculate $\left.\frac{d^k}{dx^k}\right|_{x=\pi/2} \cos x$),

   (b) using the identity $\cos x = \cos(x - \pi/2 + \pi/2)$ and the Taylor series for $\sin x$ centered at $a = 0$. [You may assume that $\sin x$ is equal to its Taylor series.]