MATH 2300-004 QUIZ 11 [in-class portion] Name: ______________________

Recall Newton's law of cooling: the rate of change in temperature of an object is proportional to the difference in temperature between the object and its surroundings,

$$\frac{dT}{dt} = k(T - T_s),$$

where $T(t)$ is temperature as a function of time, k is the proportionality constant, and T_s is the constant surrounding temperature.

Suppose a cup of coffee is 200°F when it is poured and has cooled to 190°F after one minute in a room at 70°F. When will the coffee reach 150°F? What will the temperature of the coffee be after it sits for 30 minutes?

Solution. First of all, the general solution to the differential equation above is

$$T(t) = T_s + (T(0) - T_s)e^{kt},$$

which can be obtained by integrating and solving for T

$$\begin{aligned} \int \frac{dT}{T - T_s} &= \int k\ dt, \\ \ln|T - T_s| &= kt + C_1, \\ T &= T_s + C_2 e^{kt}, \end{aligned}$$

and evaluating at $t = 0$ to get the appropriate value of C_2

$$T(0) = T_s + C_2,\ C_2 = T(0) - T_s.$$

The initial condition $T(0) = 200$ gives us

$$T(t) = 70 + 130e^{kt},$$

while the information $T(1) = 190$ (time in minutes) gives us the value of k:

$$190 = T(1) = 70 + (200 - 70)e^{k \cdot 1},\ k = \ln(12/13) \approx -0.08.$$

We now know everything about T, for instance when the temperature will be 150°F

$$T(t) = 150 = 70 + 130e^{kt},\ t = \frac{\ln(8/13)}{\ln(12/13)} \approx 6.065 \text{ minutes},$$

and what the temperature will be after 30 minutes

$$T(30) = 70 + 130e^{k \cdot 30} \approx 81.7°\text{F}.$$