MATH 2300-004 QUIZ 10 Name: ____________________________

Due Monday, April 15th at the beginning of class.

1. Consider the power series
$$f(x) = \sum_{n=1}^{\infty} \frac{2^n}{\sqrt{n}}(x-1)^n.$$
   (a) Find the interval of convergence of $f(x)$.
   (b) Differentiate f term-by-term and find the interval of convergence for the resulting power series.
   (c) Integrate f term-by-term and find the interval of convergence for the resulting power series.

2. Find a power series representation (centered at zero) for
$$\frac{1}{(1+x^3)^2},$$
(perhaps starting with the geometric series).

3. Solve the following initial value problems (explicitly for y as a function of x).
   (a) $y' + y^2 \sin x = 0,\ y(0) = -1/2$
   (b) $y' = \dfrac{x^2}{y(1+x^3)},\ y(0) = -1$

4. Suppose $y(x)$ is the solution to the initial value problem
$$y' = x^2 - y^2,\ y(0) = 1.$$
Use Euler's method (starting at $x = 0$ and with step size 0.1) to approximate $y(0.5)$.

5. Use the third degree Taylor polynomial (centered at zero) for $f(x) = \ln(1+x)$ to estimate $\ln(2)$ and use Taylor's inequality to give bounds on the error.

The next two problems are extra-credit. Point awarded for them will be added to your quiz score (although the maximum score is still only 10/10).

1. In this problem, you will show that Euler's method converges to an actual solution of the initial value problem below as you take smaller and smaller step sizes.
   (a) Use Euler's method to obtain an estimate $E_n(x)$ of the solution to
$$y' = y,\ y(0) = 1,$$
   at x by breaking up the interval between 0 and x into n equal pieces.
   (b) Find the limit as n approaches infinity in your previous answer, i.e. find
$$E(x) := \lim_{n\to\infty} E_n(x).$$
   (c) Show that the limit $E(x)$ above satisfies the initial value problem.

2. Solve the following initial value problem using power series

$$y'' + y = 0, \; y(0) = 0, \; y'(0) = 1,$$

i.e. assume $y = \sum_{n=0}^{\infty} c_n x^n$ is a solution (where the coefficients c_n are the unknowns!) and solve for the c_n recursively. Do you recognize your solution?