

Integrating trigonometric functions

Here are the most basic trigonometric identities

$$\sin^2 x + \cos^2 x = 1 \text{ (Pythagorean identity)}$$

$$\sin(x + y) = \sin x \cos y + \cos x \sin y$$

$$\cos(x + y) = \cos x \cos y - \sin x \sin y.$$

From these follow

$$\tan^2 x + 1 = \sec^2 x$$

$$1 + \cot^2 x = \csc^2 x$$

$$\sin(2x) = 2 \sin x \cos x$$

$$\cos(2x) = \cos^2 x - \sin^2 x.$$

and

$$\cos^2 x = \frac{1 + \cos(2x)}{2}, \quad \sin^2 x = \frac{1 - \cos(2x)}{2}.$$

We can use these along with our other integration techniques to integrate new functions. [Appendix C of the text contains a review of trigonometry.]

1. Integrate $\int \sin x \cos x \, dx$ two ways, with a substitution and with a trig identity.

2. $\int \cos^5 x \sin^4 x \, dx$

3. $\int \sin^3 x \, dx$

4. $\int \sec^4 x \tan^3 x \, dx$

5. $\int \sec^2 x \tan^3 x \, dx$ (problem differs from in-class handout)

6. $\int \cot^3 x \csc^3 x \, dx$ (problem differs from in-class handout)