Determine whether the following series converge or diverge. If a series converges and the terms are not eventually positive, determine whether or not the convergence is absolute.

1. $\displaystyle\sum_{n=1}^{\infty} \frac{n}{n^3+1}$

   Converges by direct or limit comparison to $\sum_n \frac{1}{n^2}$.

2. $\displaystyle\sum_{n=1}^{\infty} \frac{n^2+1}{n^3+1}$

   Diverges by limit comparison to $\sum_n \frac{1}{n}$.

3. $\displaystyle\sum_{n=1}^{\infty} \frac{n^3}{5^n}$

   Converges by the ratio test.

4. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n+1}}$

   Converges (conditionally) by the alternating series test.

5. $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n\sqrt{\ln n}}$

   Diverges by the integral test, comparing to $\int_2^{\infty} \frac{dx}{x\sqrt{\ln x}}$.

6. $\displaystyle\sum_{n=1}^{\infty} \ln\left(\frac{n}{3n+1}\right)$

   Diverges by the divergence test, $\ln\left(\frac{n}{3n+1}\right) \to \ln(1/3)$.

7. $\displaystyle\sum_{n=1}^{\infty} (-1)^{n-1} \frac{\sqrt{n}}{n+1}$

   Converges (conditionally) by the alternating series test.

8. $\displaystyle\sum_{n=1}^{\infty} \frac{\cos(3n)}{1+(1.2)^n}$

   Converges (absolutely) by direct comparison to $\sum_n \frac{1}{(1.2)^n}$.

9. $\displaystyle\sum_{n=1}^{\infty} \frac{1 \cdot 3 \cdot 5 \cdot \cdots \cdot (2n-1)}{5^n n!}$

   Converges by the ratio test.

10. $\displaystyle\sum_{n=1}^{\infty} \left(\frac{1+n}{3n}\right)^n$

    Converges by the ratio test.

11. $\sum_{n=1}^{\infty}(1-\cos(1/n))$ [Hint: compare to $\sum_n \frac{1}{n^2}$.]

Converges by limit comparison to $\sum_n \frac{1}{n^2}$.

12. $\sum_{n=1}^{\infty}\frac{8^n}{n!}$

Converges by the ratio test.

13. $\sum_{n=1}^{\infty}\frac{(-1)^{n-1}2^n}{n^2}$

Diverges by the divergence test (or ratio test).

14. $\sum_{n=1}^{\infty}\frac{\cos(n\pi)}{n}$

Converges (conditionally) by the alternating series test.

15. $\sum_{n=1}^{\infty}\frac{\tan(1/n)}{n^{3/2}}$

Converges by direct or limit comparison to $\frac{\pi}{4}\sum_n \frac{1}{n^{3/2}}$.

16. $\sum_{n=1}^{\infty}\frac{(-1)^n}{2+\sin n}$

Diverges by the divergence test, $\frac{1}{2+\sin n} \geq 1/3$.

17. $\sum_{n=1}^{\infty}\sin(1/n^2)$

Converges by limit comparison to $\sum_n \frac{1}{n^2}$.

18. $\sum_{n=1}^{\infty}\cos(1/n^2)$

Diverges by the divergence test ($\cos(1/n^2) \to 1$).

19. $\sum_{n=1}^{\infty}\tan(1/n^2)$

Converges by limit comparison to $\sum_n \frac{1}{n^2}$.

20. $\sum_{n=1}^{\infty}ne^{-n^2}$

Converges by the integral test or the ratio test.